

Stepen i kvadratni koren

20. Израчунати:

$$\text{A) } 11 \cdot 3^2 - \frac{9^2}{3};$$

$$\text{B) } (3 \cdot 2)^2 - \frac{1}{2} \cdot 4^2.$$

A)

$$\begin{aligned} 11 \cdot 3^2 - \frac{9^2}{3} &= 11 \cdot 9 - \frac{81}{3} \\ &= 99 - 27 \\ &= 72 \end{aligned}$$

B)

$$\begin{aligned} (3 \cdot 2)^2 - \frac{1}{2} \cdot 4^2 &= 6^2 - \frac{1}{2} \cdot 16 \\ &= 36 - 8 \\ &= 28 \end{aligned}$$

21. Израчунати $(2^3 \cdot 3^2) : 6 + (2^3 - 3^2) \cdot 10^2$.

$$(2^3 \cdot 3^2) : 6 + (2^3 - 3^2) \cdot 10^2 =$$

$$(8 \cdot 9) : 6 + (8 - 9) \cdot 100 =$$

$$72 : 6 + (-1) \cdot 100 = 12 - 100 = -88$$

22. Израчунати:

$$\text{A) } \frac{5}{3^2} + \left(\frac{2}{3}\right)^3 - \frac{4^2}{9};$$

$$\text{B) } \left(-\frac{3}{5}\right)^3 \cdot (12 - 17)^2.$$

A)

$$\begin{aligned} \frac{5}{3^2} + \left(\frac{2}{3}\right)^3 - \frac{4^2}{9} &= \frac{5^{(3)}}{9} + \frac{8}{27} - \frac{16^{(3)}}{9} = \\ &= \frac{15 + 8 - 48}{27} = -\frac{25}{27} \end{aligned}$$

B)

$$\left(-\frac{3}{5}\right)^3 \cdot (12 - 17)^2 = -\frac{27}{125} \cdot (-5)^2$$

$$= -\frac{27}{125} \cdot 25$$

$$= -\frac{27}{5}$$

23. Израчунати $\frac{2^3 \cdot 4^2}{16^2 : 8}$.

$$\frac{2^3 \cdot 4^2}{16^2 : 8} =$$

Pazi, ovde ne možemo odmah upotrebiti pravila za stepenovanje jer osnove nisu iste! Zato ćemo uraditi:

$$4 = 2^2, 16 = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4, 8 = 2 \cdot 2 \cdot 2 = 2^3$$

Vratimo se u zadatak:

$$\frac{2^3 \cdot 4^2}{16^2 : 8} = \frac{2^3 \cdot (2^2)^2}{(2^4)^2 : 2^3} = \frac{2^3 \cdot 2^4}{2^8 : 2^3} = \frac{2^{3+4}}{2^{8-3}} = \frac{2^7}{2^5} = 2^{7-5} = 2^2 = 4$$

24. Израчунати $2^5 \cdot \frac{4^3 \cdot 8}{4^2 \cdot 2^6} - 2^3 \cdot \frac{16^2 \cdot 2^4}{2^3 \cdot 8^3}$.

$$2^5 \cdot \frac{4^3 \cdot 8}{4^2 \cdot 2^6} - 2^3 \cdot \frac{16^2 \cdot 2^4}{2^3 \cdot 8^3} =$$

$$2^5 \cdot \frac{(2^2)^3 \cdot 2^3}{(2^2)^2 \cdot 2^6} - 2^3 \cdot \frac{(2^4)^2 \cdot 2^4}{2^3 \cdot (2^3)^3} =$$

$$\frac{2^5 \cdot \cancel{2^6} \cdot 2^3}{1 \cdot 2^4 \cdot \cancel{2^6}} - \frac{\cancel{2^3} \cdot 2^8 \cdot 2^4}{1 \cdot \cancel{2^3} \cdot 2^9} =$$

$$\frac{2^5 \cdot 2^3}{2^4} - \frac{2^8 \cdot 2^4}{2^9} =$$

$$\frac{2^8}{2^4} - \frac{2^{12}}{2^9} = 2^4 - 2^3 = 16 - 8 = 8$$

25. Израчунати $\frac{81^2 \cdot 3^2 : 27}{3^5 \cdot 9}$.

$$\frac{81^2 \cdot 3^2 : 27}{3^5 \cdot 9} =$$

Opet sličan problem, moramo ‘napraviti’ iste osnove!

$$81 = 3 \cdot 3 \cdot 3 \cdot 3 = 3^4, 27 = 3 \cdot 3 \cdot 3 = 3^3, 9 = 3^2$$

Sada je:

$$\frac{81^2 \cdot 3^2 : 27}{3^5 \cdot 9} = \frac{(3^4)^2 \cdot 3^2 : 3^3}{3^5 \cdot 3^2} = \frac{3^8 \cdot 3^2 : 3^3}{3^7} = \frac{3^{8+2-3}}{3^7} = \frac{3^7}{3^7} = 1$$

26. Израчунати $3^4 + 36^6 + 15^7 - 6^{12} - 3^7 \cdot 5^7$.

$$3^4 + 36^6 + 15^7 - 6^{12} - 3^7 \cdot 5^7 =$$

$$3^4 + (6^2)^6 + 15^7 - 6^{12} - (3 \cdot 5)^7 =$$

$$3^4 + \overbrace{6^{12} + 15^7 - 6^{12} - 15^7}^{(potiru se!)} =$$

$$= 3^4 = 81$$

27. Израчунати:

A) $\frac{3^7 + 3^5}{3^7 - 3^5}$;

Б) $\frac{2^8 - 2^5}{2^8 + 2^5}$.

A)

U ovom zadatku ćemo upotrebiti distributivni zakon, odnosno izvući ćemo ‘zajednički’ ispred zagrade!

$$A) \frac{3^7 + 3^5}{3^7 - 3^5} = \frac{3^5 \cdot 3^2 + 3^5}{3^5 \cdot 3^2 - 3^5} = \frac{3^5(3^2 + 1)}{3^5(3^2 - 1)} = \frac{9 + 1}{9 - 1} = \frac{10}{8} = \frac{5}{4}$$

$$B) \frac{2^8 - 2^5}{2^8 + 2^5} = \frac{2^5 \cdot 2^3 - 2^5}{2^5 \cdot 2^3 + 2^5} = \frac{2^5(2^3 - 1)}{2^5(2^3 + 1)} = \frac{8 - 1}{8 + 1} = \frac{7}{9}$$

28. Упростити изразе:

A) $12a^6b^3c : (-3a^2b)$ ($a \neq 0, b \neq 0$);

Б) $-4a^3b^3c : (2ab^2c)$ ($a \neq 0, b \neq 0, c \neq 0$).

A) $12a^6b^3c : (-3a^2b) =$

Kako ovde razmišljati?

Uočiti brijke, a -ove i... b -ove

$$12a^6b^3c : (-3a^2b) =$$

$$= [12 : (-3)] \cdot [a^6 : a^2] \cdot [b^3 : b] \cdot c$$

$$= -4 \cdot a^4 \cdot b^2 \cdot c$$

B)

$$\begin{aligned} & -4a^3b^3c : (2ab^2c) = \\ & = [4 : 2] \cdot [a^3 : a] \cdot [b^3 : b^2] \cdot [c : c] \\ & = -2 \cdot a^2 \cdot b^1 \cdot 1 \\ & = -2a^2b \end{aligned}$$

29. Упростити израз $\frac{(x^4)^3 \cdot x^3 : x^5}{(x^5 : x^2)^3}$ ($x \neq 0$).

$$\frac{(x^4)^3 \cdot x^3 : x^5}{(x^5 : x^2)^3} = \frac{x^{12} \cdot x^3 : x^5}{(x^{5-2})^3} = \frac{x^{12+3-5}}{(x^3)^3} = \frac{x^{10}}{x^9} = x^{10-9} = x^1 = x$$

30. Упростити израз $\left(\frac{(2a^3)^2}{2a^4b^3}\right)^3$ ($a \neq 0, b \neq 0$).

$$\left(\frac{(2a^3)^2}{2a^4b^3}\right)^3 = \left(\frac{4a^6}{2a^4b^3}\right)^3 = \left(\frac{2a^2}{b^3}\right)^3 = \frac{8a^6}{b^9}$$

31. Израчунати вредност израза $\frac{(-2)^3}{-2^2} - \frac{(-2)^2}{-2^3}$.

$$\frac{(-2)^3}{-2^2} - \frac{(-2)^2}{-2^3} = \frac{-8}{-4} - \frac{4}{-8} = 2 + \frac{1}{2} = 2\frac{1}{2} = \frac{5}{2}$$

32. Упростити изразе:

А) $(-a)^2 \cdot (-a)^3 \cdot (-a)^5$;

Б) $(-x)^2 \cdot (-x^2) \cdot (-x^3) \cdot (-x)^3$.

А) $(-a)^2 \cdot (-a)^3 \cdot (-a)^5 = (-a)^{2+3+5} = (-a)^{10} = a^{10}$

Б) $(-x)^2 \cdot (-x^2) \cdot (-x^3) \cdot (-x)^3 = \text{pažljivo}$
 $= x^2 \cdot (-x^2) \cdot (-x^3) \cdot (-x^3) = [\text{ima tri minusa koji daju opet } -]$
 $= -x^{2+2+3+3} = -x^{10}$

33. Упростити изразе:

А) $\frac{3^{6n} \cdot 3^{2n+1}}{3^{3n} \cdot 3^{5n}}$;

Б) $\frac{8^{2n-1}}{2^{6n-5}}$.

$$A) \frac{3^{6n} \cdot 3^{2n+1}}{3^{3n} \cdot 3^{5n}} = \frac{3^{6n+2n+1}}{3^{3n+5n}} = \frac{3^{8n+1}}{3^{8n}} = 3^{8n+1-8n} = 3^1 = 3$$

$$B) \frac{8^{2n-1}}{2^{6n-5}} = \frac{(2^3)^{2n-1}}{2^{6n-5}} = \frac{2^{6n-3}}{2^{6n-5}} = \text{pazi na minus!!!}$$

$$= 2^{6n-3-(6n-5)} = 2^{6n-3-6n+5} = 2^2 = 4$$

34. Израчунати вредност израза

$$A) \sqrt{1 - \frac{9}{25}} + \sqrt{1 + \frac{39}{25}};$$

$$B) \sqrt{\frac{1}{4}} - \sqrt{\frac{1}{(-2)^2}}.$$

$$A) 5\sqrt{2} + 3\sqrt{8} - \sqrt{50} - \sqrt{98} = [\text{pazi, moramo najpre da sredimo izraze unutar korena}]$$

$$\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$$

$$\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$$

$$\sqrt{98} = \sqrt{49 \cdot 2} = \sqrt{49} \cdot \sqrt{2} = 7\sqrt{2}$$

Vratimo se sad na zadatak:

$$\cancel{5\sqrt{2}} + 3 \cdot 2\sqrt{2} - \cancel{5\sqrt{2}} - 7\sqrt{2} = 6\sqrt{2} - 7\sqrt{2} = -\sqrt{2}$$

35. Упростити израз $5\sqrt{2} + 3\sqrt{8} - \sqrt{50} - \sqrt{98}$.

$$A) 2\sqrt{2} + \sqrt{72} - 3\sqrt{8} = 2\sqrt{2} + \sqrt{36 \cdot 2} - 3 \cdot \sqrt{4 \cdot 2}$$

$$= 2\sqrt{2} + 6\sqrt{2} - 3 \cdot 2\sqrt{2}$$

$$= 2\sqrt{2} + 6\sqrt{2} - 6\sqrt{2}$$

$$= 2\sqrt{2}$$

$$B) -3\sqrt{(-2)^2} + (2\sqrt{3})^2 - (4\sqrt{5})^2 =$$

$$-3\sqrt{4} + 2^2 \cdot \sqrt{3}^2 - 4^2 \sqrt{5}^2 =$$

$$-3 \cdot 2 + 4 \cdot 3 - 16 \cdot 5 = -6 + 12 - 80 = -74$$

36. Упростити изразе:

$$A) 2\sqrt{2} + \sqrt{72} - 3\sqrt{8};$$

$$B) -3\sqrt{(-2)^2} + (2\sqrt{3})^2 - (4\sqrt{5})^2.$$

37. Упростити изразе:

А) $\frac{3}{\sqrt{7}} - \sqrt{28}$;

Б) $3\sqrt{3} + \sqrt{108} - 4\sqrt{27}$.

$$\begin{aligned} \text{А) } \frac{3}{\sqrt{7}} - \sqrt{28} &= \frac{3}{\sqrt{7}} - \frac{\sqrt{7}}{\sqrt{7}} - \sqrt{4 \cdot 7} \\ &= \frac{3\sqrt{7}}{\sqrt{7}} - 2\sqrt{7} \\ &= \frac{3\sqrt{7} - 14\sqrt{7}}{7} = -\frac{11\sqrt{7}}{7} \end{aligned}$$

Б) $3\sqrt{3} + \sqrt{108} - 4\sqrt{27} =$

$$3\sqrt{3} + \sqrt{36 \cdot 3} - 4\sqrt{9 \cdot 3} =$$

$$3\sqrt{3} + 6\sqrt{3} - 4 \cdot 3\sqrt{3} =$$

$$3\sqrt{3} + 6\sqrt{3} - 12\sqrt{3} = -3\sqrt{3}$$