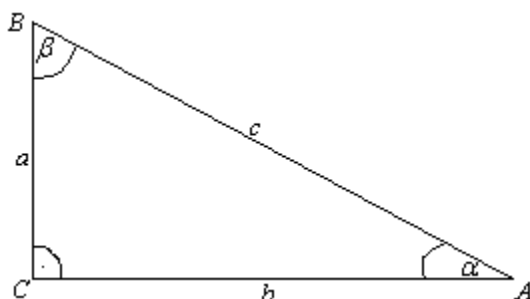


TRIGONOMETRIJSKE FUNKCIJE OŠTROG UGLA

Trigonometrija je prvobitno predstavlja oblast matematike koje se bavila izračunavanjem nepoznatih elemenata trougla pomoću poznatih. Sam njen naziv potiče od dve grčke reči TRIGONOS- što znači trougao i METRON- što znači mera. Kako se definišu trigonometrijske funkcije?

Posmatrajmo pravougli trougao ABC.



$a, b \rightarrow$ katete
 $c \rightarrow$ hipotenuza
 $a^2 + b^2 = c^2 \rightarrow$ Pitagorina teorema

$$\sin \alpha = \frac{\text{naspramna kateta}}{\text{hipotenuza}} = \frac{a}{c}$$

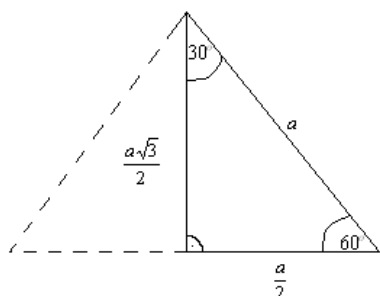
$$\cos \alpha = \frac{\text{nalegla kateta}}{\text{hipotenuza}} = \frac{b}{c}$$

$$\operatorname{tg} \alpha = \frac{\text{naspramna kateta}}{\text{nalegla kateta}} = \frac{a}{b}$$

$$\operatorname{ctg} \alpha = \frac{\text{nalegla kateta}}{\text{naspramna kateta}} = \frac{b}{a}$$

PAZI: Sam simbol sin, cos, tg, ctg sam za sebe ne označava nikakvu veličinu!!! Uvek mora da ima i ugao.

Izračunajmo vrednost trigonometrijskih funkcija za uglove od $30^\circ, 45^\circ$ i 60° . Najpre ćemo posmatrati polovinu jednakostraničnog trougla.



Kao što znamo visina jednakostraničnog trougla je

$$h = \frac{a\sqrt{3}}{2}$$

$$\sin 30^\circ = \frac{\text{naspramna kateta}}{\text{hipotenuza}} = \frac{\frac{a}{2}}{a} = \frac{a}{2a} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\text{nalegla kateta}}{\text{hipotenuza}} = \frac{\frac{a\sqrt{3}}{2}}{a} = \frac{\sqrt{3}}{2}$$

$$\operatorname{tg} 30^\circ = \frac{\text{naspramna kateta}}{\text{nalegla kateta}} = \frac{\frac{a}{2}}{\frac{a\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} \text{ (racionališemo)} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\operatorname{ctg} 30^\circ = \frac{\text{nalegla kateta}}{\text{naspramna kateta}} = \frac{\frac{a\sqrt{3}}{2}}{\frac{a}{2}} = \sqrt{3}$$

Sada ćemo uraditi (po definiciji) i za ugao od 60° .

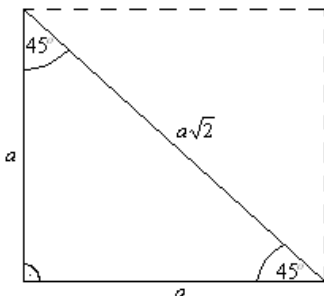
$$\sin 60^\circ = \frac{\frac{a\sqrt{3}}{2}}{a} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{\frac{a}{2}}{a} = \frac{1}{2}$$

$$\operatorname{tg} 60^\circ = \frac{\frac{a\sqrt{3}}{2}}{\frac{a}{2}} = \sqrt{3}$$

$$\operatorname{ctg} 60^\circ = \frac{\frac{a}{2}}{\frac{a\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$$

Za vrednost trigonometrijskih funkcija ugla od 45° upotrebićemo polovinu kvadrata.



Kao što znamo dijagonala kvadrata je $d = a\sqrt{2}$

$$\sin 45^\circ = \frac{\text{naspramna kateta}}{\text{hipotenuza}} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{\text{nalegla kateta}}{\text{hipotenuza}} = \frac{a}{a\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\text{tg} 45^\circ = \frac{\text{naspramna kateta}}{\text{nalegla kateta}} = \frac{a}{a} = 1$$

$$\text{ctg} 45^\circ = \frac{\text{nalegla kateta}}{\text{naspramna kateta}} = \frac{a}{a} = 1$$

Na ovaj način smo dobili tablicu:

	$\alpha = 30^\circ$	$\alpha = 45^\circ$	$\alpha = 60^\circ$
$\sin \alpha$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos \alpha$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\text{tg} \alpha$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
$\text{ctg} \alpha$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$

Naravno, kasnije ćemo tablicu proširiti na sve uglove od $0^\circ \rightarrow 360^\circ$.

Osnovni trigonometrijski identiteti:

$$1) \sin^2 \alpha + \cos^2 \alpha = 1$$

$$2) \text{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$3) \text{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$4) \text{tg} \alpha \cdot \text{ctg} \alpha = 1$$

Da probamo da dokažemo neke od identiteta:

$$1) \sin^2 \alpha + \cos^2 \alpha = (\text{pogledajmo definicije: } \sin \alpha = \frac{a}{c} \text{ i } \cos \alpha = \frac{b}{c}; \text{ to da zapamtimo}) =$$

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2 + b^2}{c^2} = (\text{važi Pitagorina teorema, } a^2 + b^2 = c^2) = \frac{c^2}{c^2} = 1$$

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$$2) \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{a \cdot c}{b \cdot c} = \frac{a}{b} = \operatorname{tg} \alpha \text{ slično se dokazuje i za } \operatorname{ctg} \alpha$$

$$4) \operatorname{tg} \alpha \cdot \operatorname{ctg} \alpha = (\text{zamenimo iz definicije, da je } \operatorname{tg} \alpha = \frac{a}{b} \text{ i } \operatorname{ctg} \alpha = \frac{b}{a}) = \frac{a}{b} \cdot \frac{b}{a} = 1$$

Baš lako, zar ne?

Iz osnovnih identiteta se mogu izvesti razne druge jednakosti:

1) Ako krenemo od:

$$\sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow \text{ovo delimo sa } \cos^2 \alpha$$

$$\frac{\sin^2 \alpha}{\cos^2 \alpha} + \frac{\cos^2 \alpha}{\cos^2 \alpha} = \frac{1}{\cos^2 \alpha}$$

$$\operatorname{tg}^2 \alpha + 1 = \frac{1}{\cos^2 \alpha} \rightarrow \text{Oдавde izrazimo } \cos^2 \alpha$$

$$\boxed{\cos^2 \alpha = \frac{1}{\operatorname{tg}^2 \alpha + 1}}$$

Ako sad ovo zamenimo u:

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin^2 \alpha + \frac{1}{\operatorname{tg}^2 \alpha + 1} = 1$$

$$\sin^2 \alpha = 1 - \frac{1}{\operatorname{tg}^2 \alpha + 1}$$

$$\sin^2 \alpha = \frac{\operatorname{tg}^2 \alpha + 1 - 1}{\operatorname{tg}^2 \alpha + 1}$$

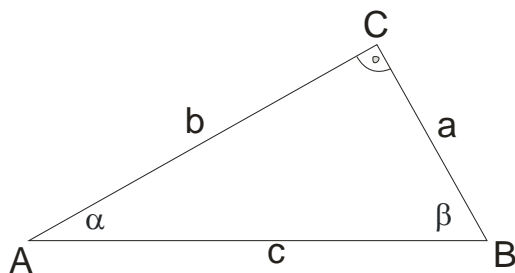
$$\boxed{\sin^2 \alpha = \frac{\operatorname{tg}^2 \alpha}{\operatorname{tg}^2 \alpha + 1}}$$

Ove dve identičnosti ćemo zapisati i koristiti ih u zadacima!!!

Još jedna stvar, da izvedemo i trigonometrijske funkcije komplementnog ugla. Kako je kod pravouglog trougla $\alpha + \beta = 90^\circ$ tj. komplementni su, važi:

$$\begin{array}{ll} \sin(90^\circ - \alpha) = \cos \alpha & \text{tj.} \quad \sin \beta = \cos \alpha \\ \cos(90^\circ - \alpha) = \sin \alpha & \cos \beta = \sin \alpha \\ \operatorname{tg}(90^\circ - \alpha) = \operatorname{ctg} \alpha & \operatorname{tg} \beta = \operatorname{ctg} \alpha \\ \operatorname{ctg}(90^\circ - \alpha) = \operatorname{tg} \alpha & \operatorname{ctg} \beta = \operatorname{tg} \alpha \end{array}$$

Odakle ovo?



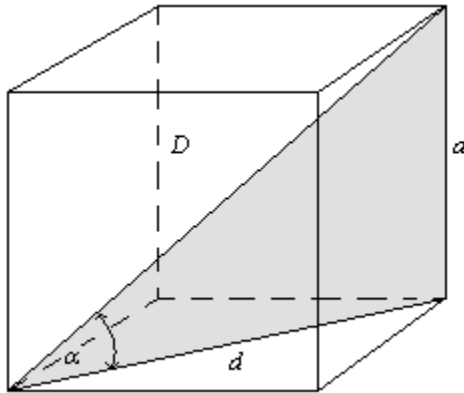
sa slike (po definiciji) je

$$\begin{array}{ll} \sin \alpha = \frac{a}{c} & \sin \beta = \frac{b}{c} \\ \cos \alpha = \frac{b}{c} & \cos \beta = \frac{a}{c} \\ \operatorname{tg} \alpha = \frac{a}{b} & \operatorname{tg} \beta = \frac{b}{a} \\ \operatorname{ctg} \alpha = \frac{b}{a} & \operatorname{ctg} \beta = \frac{a}{b} \end{array}$$

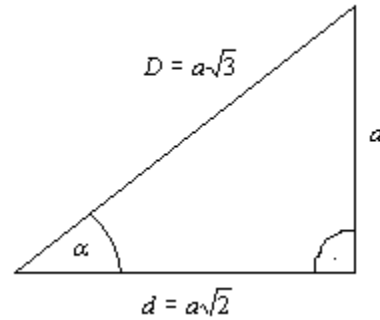
1) Date su katete pravouglog trougla $a=8\text{cm}$ i $b=6\text{cm}$. Odrediti vrednost svih trigonometrijskih funkcija uglova α i β

$$\begin{array}{ll} a = 8\text{cm} & \\ b = 6\text{cm} & \\ \hline c^2 = a^2 + b^2 & \sin \alpha = \frac{a}{c} = \frac{8}{10} = \frac{4}{5} = \cos \beta \\ c^2 = 8^2 + 6^2 & \cos \alpha = \frac{b}{c} = \frac{6}{10} = \frac{3}{5} = \sin \beta \\ c^2 = 64 + 36 & \operatorname{tg} \alpha = \frac{a}{b} = \frac{8}{6} = \frac{4}{3} = \operatorname{ctg} \beta \\ c^2 = 100 & \operatorname{ctg} \alpha = \frac{b}{a} = \frac{6}{8} = \frac{3}{4} = \operatorname{tg} \beta \\ c = 10\text{cm} & \end{array}$$

2) Izračunati vrednost trigonometrijskih funkcija nagibnog ugla dijagonale kocke prema osnovi.



Izvučemo na stranu ovaj trougao:



Kao što znamo mala dijagonala je $d = a\sqrt{2}$, a velika dijagonala (telesna) $D = a\sqrt{3}$. Po definicijama je:

$$\sin \alpha = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\cos \alpha = \frac{a\sqrt{2}}{a\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

$$\operatorname{tg} \alpha = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\operatorname{ctg} \alpha = \frac{a\sqrt{2}}{a} = \sqrt{2}$$

3) U pravouglom trouglu je $c = 24\text{cm}$ i $\sin \alpha = 0,8$. Odrediti katete.

$$c = 24\text{cm}$$

$$\sin \alpha = 0,8$$

$$a = ?$$

$$b = ?$$

Po definiciji je:

$$\sin \alpha = \frac{a}{c}$$

$$0,8 = \frac{a}{24}$$

$$a = 24 \cdot 0,8$$

$$a = 19,2\text{cm}$$

$b^2 = c^2 - a^2$ sad ide Pitagorina teorema

$$b^2 = 24^2 - (19,2)^2$$

$$b^2 = 576 - 368,64$$

$$b^2 = 207,36$$

$$b = 14,4\text{cm}$$

4) Izračunati vrednost ostalih trigonometrijskih funkcija ako je:

a) $\sin \alpha = 0,6$

b) $\cos \alpha = \frac{12}{13}$

v) $\operatorname{tg} \alpha = 0,225$

Rešenje:

a) $\sin \alpha = \frac{3}{5}$ jer $0,6 = \frac{6}{10} = \frac{3}{5}$. Najpre ćemo iskoristiti da je $\sin^2 \alpha + \cos^2 \alpha = 1$

$$\left(\frac{3}{5}\right)^2 + \cos^2 \alpha = 1$$

$$\cos^2 \alpha = 1 - \frac{9}{25}$$

$$\cos^2 \alpha = \frac{16}{25}$$

$$\cos \alpha = \pm \sqrt{\frac{16}{25}}$$

$$\cos \alpha = \pm \frac{4}{5}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\operatorname{ctg} \alpha = \frac{1}{\operatorname{tg} \alpha} = \frac{4}{3}$$

Pošto su oštri uglovi u pitanju:

$$\boxed{\cos \alpha = +\frac{4}{5}}$$

b)

$$\cos \alpha = \frac{12}{13}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin^2 \alpha + \left(\frac{12}{13}\right)^2 = 1$$

$$\sin^2 \alpha = 1 - \frac{144}{169}$$

$$\sin^2 \alpha = \frac{25}{169}$$

$$\sin \alpha = \pm \sqrt{\frac{25}{169}}$$

$$\sin \alpha = \pm \frac{5}{13}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$$

$$\operatorname{ctg} \alpha = \frac{12}{5}$$

oštar ugao, pa uzimamo +

$$\boxed{\sin \alpha = \frac{5}{13}}$$

$$v) \operatorname{tg} \alpha = 0,225 = \frac{225}{1000} = \frac{9}{40}$$

Iskoristićemo jednakosti: $\boxed{\sin^2 \alpha = \frac{\operatorname{tg}^2 \alpha}{\operatorname{tg}^2 \alpha + 1}}$ i $\boxed{\cos^2 \alpha = \frac{1}{\operatorname{tg}^2 \alpha + 1}}$

$$\sin^2 \alpha = \frac{\operatorname{tg}^2 \alpha}{\operatorname{tg}^2 \alpha + 1} \qquad \sin^2 \alpha = \frac{81}{1681}$$

$$\sin^2 \alpha = \frac{\left(\frac{9}{40}\right)^2}{\left(\frac{9}{40}\right)^2 + 1} \qquad \sin \alpha = \pm \sqrt{\frac{81}{1681}}$$

$$\sin \alpha = \pm \frac{9}{41}$$

$$\sin \alpha = +\frac{9}{41}$$

$$\sin^2 \alpha = \frac{\frac{81}{1600}}{\frac{81}{1600} + 1}$$

$$\sin^2 \alpha = \frac{\frac{81}{1600}}{\frac{81}{1600} + 1} = \frac{81}{81 + 1600} = \frac{81}{1681}$$

$$\cos^2 \alpha = \frac{1}{\operatorname{tg}^2 \alpha + 1}$$

$$\cos^2 \alpha = \frac{1}{\frac{81}{1600} + 1} \rightarrow \cos^2 \alpha = \frac{1600}{1681}$$

$$\cos \alpha = \pm \sqrt{\frac{1600}{1681}} \rightarrow \cos \alpha = \pm \frac{40}{41}$$

$$\cos \alpha = +\frac{40}{41}$$

Za kotangens je lako:

$$\operatorname{ctg} \alpha = \frac{1}{\operatorname{tg} \alpha}$$

$$\operatorname{ctg} \alpha = \frac{40}{9}$$

5) Izračunaj vrednosti ostalih trigonometrijskih funkcija ako je:

$$\text{a) } \sin \alpha = \frac{a^2 - 9}{a^2 + 9}$$

$$\text{b) } \text{ctg} \alpha = \frac{a^2 - 4}{4a}$$

a)

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\cos^2 \alpha = 1 - \sin^2 \alpha$$

$$\cos^2 \alpha = 1 - \left(\frac{a^2 - 9}{a^2 + 9} \right)^2$$

$$\cos^2 \alpha = 1 - \frac{(a^2 - 9)^2}{(a^2 + 9)^2}$$

$$\cos^2 \alpha = \frac{(a^2 + 9)^2 - (a^2 - 9)^2}{(a^2 + 9)^2}$$

$$\cos^2 \alpha = \frac{a^4 + 18a^2 + 81 - a^4 + 18a^2 - 81}{(a^2 + 9)^2}$$

$$\cos^2 \alpha = \frac{36a^2}{(a^2 + 9)^2}$$

$$\cos \alpha = \sqrt{\frac{36a^2}{(a^2 + 9)^2}}$$

$$\cos \alpha = \frac{6a}{a^2 + 9}$$

$$\text{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\frac{a^2 - 9}{\cancel{a^2 + 9}}$$

$$\text{tg} \alpha = \frac{6a}{\cancel{a^2 + 9}}$$

$$\text{tg} \alpha = \frac{a^2 - 9}{6a}$$

$$\text{ctg} \alpha = \frac{6a}{a^2 - 9}$$

$$\text{b) } \text{ctg} \alpha = \frac{a^2 - 4}{4a} \Rightarrow \text{tg} \alpha = \frac{4a}{a^2 - 4}$$

$$\sin^2 \alpha = \frac{\text{tg}^2 \alpha}{\text{tg}^2 \alpha + 1}$$

$$\sin^2 \alpha = \frac{\left(\frac{4a}{a^2 - 4} \right)^2}{\left(\frac{4a}{a^2 - 4} \right)^2 + 1}$$

$$\sin^2 \alpha = \frac{16a^2}{\frac{(a^2 - 4)^2}{16a^2} + 1}$$

$$\sin^2 \alpha = \frac{16a^2}{16a^2 + a^4 - 8a^2 + 16}$$

$$\sin^2 \alpha = \frac{16a^2}{a^4 + 8a^2 + 16}$$

$$\sin \alpha = \sqrt{\frac{16a^2}{(a^2 + 4)^2}}$$

$$\sin \alpha = \frac{4a}{a^2 + 4}$$

$$\cos^2 \alpha = \frac{1}{\text{tg}^2 \alpha + 1}$$

$$\cos^2 \alpha = \frac{1}{\left(\frac{4a}{a^2 - 4} \right)^2 + 1}$$

$$\cos^2 \alpha = \frac{1}{16a^2 + (a^2 - 4)^2}$$

$$\cos^2 \alpha = \frac{1}{\frac{(a^2 + 4)^2}{(a^2 - 4)^2}}$$

$$\cos^2 \alpha = \frac{(a^2 - 4)^2}{(a^2 + 4)^2}$$

$$\cos \alpha = \sqrt{\frac{(a^2 - 4)^2}{(a^2 + 4)^2}}$$

$$\cos \alpha = \frac{a^2 - 4}{a^2 + 4}$$

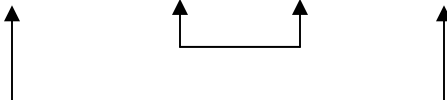
6) **Dokazati identitet** $\left(1 + \operatorname{tg}x + \frac{1}{\cos x}\right) \cdot \left(1 + \operatorname{tg}x - \frac{1}{\cos x}\right) = 2\operatorname{tg}x$

$$\begin{aligned} & \left(1 + \operatorname{tg}x + \frac{1}{\cos x}\right) \cdot \left(1 + \operatorname{tg}x - \frac{1}{\cos x}\right) = \\ & \left(1 + \frac{\sin x}{\cos x} + \frac{1}{\cos x}\right) \cdot \left(1 + \frac{\sin x}{\cos x} - \frac{1}{\cos x}\right) = \\ & \frac{\cos x + \sin x + 1}{\cos x} \cdot \frac{\cos x + \sin x - 1}{\cos x} = \text{gore je razlika kvadrata} \\ & \frac{(\cos x + \sin x)^2 - 1^2}{\cos^2 x} = (\text{jedinicu ćemo zameniti sa } \sin^2 x + \cos^2 x) \\ & \frac{\cos^2 x + 2\cos x \sin x + \sin^2 x - \sin^2 x - \cos^2 x}{\cos^2 x} = \frac{2\cancel{\cos x} \sin x}{\cos^2 x} = \\ & = 2 \frac{\sin x}{\cos x} = 2\operatorname{tg}x \end{aligned}$$

7) **Dokazati da je:**

a) $\cos^2 18^\circ + \cos^2 36^\circ + \cos^2 54^\circ + \cos^2 72^\circ = 2$

Pošto važi da kad je $\alpha + \beta = 90^\circ$ $\cos \alpha = \sin \beta$, $\cos 54^\circ$ ćemo zameniti sa $\sin 36^\circ$ a $\cos 72^\circ$ ćemo zameniti sa $\sin 18^\circ$. Onda je:

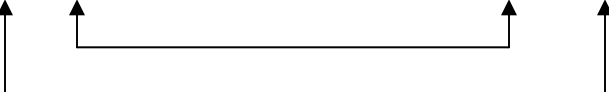
$$\begin{aligned} & \cos^2 18^\circ + \cos^2 36^\circ + \cos^2 54^\circ + \cos^2 72^\circ = \\ & \cos^2 18^\circ + \cos^2 36^\circ + \sin^2 36^\circ + \sin^2 18^\circ = \end{aligned}$$


$$= 1 + 1 = 2$$

b)

$$\operatorname{tg}1^\circ \cdot \operatorname{tg}2^\circ \cdot \operatorname{tg}3^\circ \dots \operatorname{tg}44^\circ \cdot \operatorname{tg}45^\circ \cdot \operatorname{tg}46^\circ \dots \operatorname{tg}89^\circ = 1$$

= Kako je $\operatorname{tg} \alpha = \operatorname{ctg} \beta$ ($\alpha + \beta = 90^\circ$) biće:

$$\operatorname{tg}1^\circ \cdot \operatorname{tg}2^\circ \cdot \operatorname{tg}3^\circ \dots \operatorname{tg}44^\circ \cdot \operatorname{tg}45^\circ \cdot \operatorname{ctg}44^\circ \dots \operatorname{ctg}2^\circ \cdot \operatorname{ctg}1^\circ$$


= Kako je $\operatorname{tg} \alpha \cdot \operatorname{ctg} \alpha = 1$

$$= 1 \cdot 1 \cdot \dots \cdot \operatorname{tg}45^\circ = 1$$

8) **Dokazati identitet** $\frac{3}{1 - \sin^6 \alpha - \cos^6 \alpha} = (\operatorname{tg} \alpha + \operatorname{ctg} \alpha)^2$

$$\frac{3}{1 - \sin^6 x - \cos^6 x} = \frac{3}{1 - (\sin^6 x + \cos^6 x)} = \text{Pokušaćemo da transformišemo izraz}$$

$\sin^6 x - \cos^6 x$ Podjimo od $\sin^2 x - \cos^2 x = 1$ pa ‘dignemo’ na treći stepen:

$$(A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$$

$$\sin^2 x + \cos^2 x = 1 \quad ()^3$$

$$\sin^6 x + 3\sin^4 x \cos^2 x + 3\sin^2 x \cos^4 x + \cos^6 x = 1$$

$$\sin^6 x + 3\sin^2 x \cos^2 x \underbrace{(\sin^2 x + \cos^2 x)}_1 + \cos^6 x = 1$$

Dakle: $\sin^6 x + \cos^6 x = 1 - 3\sin^2 x \cos^2 x$

Vratimo se u zadatak:

$$= \frac{3}{1 - 1 + 3\sin^2 x \cos^2 x} = \frac{3}{3\sin^2 x \cos^2 x} = \frac{1}{\sin^2 x \cos^2 x}$$

Da vidimo sad desnu stranu:

$$\begin{aligned} (\operatorname{tg} \alpha + \operatorname{ctg} \alpha)^2 &= \operatorname{tg}^2 \alpha + 2\operatorname{tg} \alpha \operatorname{ctg} \alpha + \operatorname{ctg}^2 \alpha \\ &= \frac{\sin^2 \alpha}{\cos^2 \alpha} + 2 + \frac{\cos^2 \alpha}{\sin^2 \alpha} \\ &= \frac{\sin^4 \alpha + 2\sin^2 \alpha \cos^2 \alpha + \cos^4 \alpha}{\sin^2 \alpha \cos^2 \alpha} \\ &= \frac{(\sin^2 \alpha + \cos^2 \alpha)^2}{\sin^2 \alpha \cos^2 \alpha} \\ &= \frac{1}{\sin^2 \alpha \cos^2 \alpha} \end{aligned}$$

Ovim smo dokazali da su leva i desna strana jednake:

Uslov je

$$1 - \sin^6 \alpha - \cos^6 \alpha \neq 0$$

$$\sin^6 \alpha - \cos^6 \alpha \neq 1$$

$$1 - 3\sin^2 \alpha \cos^2 \alpha \neq 1$$

$$\sin^2 \alpha \cos^2 \alpha \neq 0$$

$$\sin \alpha \neq 0 \wedge \cos \alpha \neq 0$$

9) Dokazati identitet: $(tg^3\alpha + \frac{1-tg\alpha}{ctg\alpha}) : (\frac{1-ctg\alpha}{tg\alpha} + ctg^3\alpha) = tg^4\alpha$

Kao i obično, krenemo od teže strane dok ne dodjemo da lakše...

$$(tg^3\alpha + \frac{1-tg\alpha}{ctg\alpha}) : (\frac{1-ctg\alpha}{tg\alpha} + ctg^3\alpha) =$$

$$(tg^3\alpha + \frac{1-tg\alpha}{\frac{1}{tg\alpha}}) : (\frac{1-\frac{1}{tg\alpha}}{tg\alpha} + \frac{1}{tg^3\alpha}) =$$

$$(tg^3\alpha + tg\alpha \cdot \frac{tg\alpha - 1}{1}) : (\frac{tg\alpha - 1}{tg\alpha} + \frac{1}{tg^3\alpha}) =$$

$$(tg^3\alpha + tg\alpha - tg^2\alpha) : (\frac{tg\alpha - 1}{tg^2\alpha} + \frac{1}{tg^3\alpha}) =$$

$$(tg^3\alpha - tg^2\alpha + tg\alpha) : (\frac{tg\alpha(tg\alpha - 1) + 1}{tg^3\alpha}) =$$

$$tg\alpha(tg^2\alpha - tg\alpha + 1) : (\frac{tg^2\alpha - tg\alpha + 1}{tg^3\alpha}) = \frac{tg\alpha(tg^2\alpha - tg\alpha + 1)}{1} : (\frac{tg^2\alpha - tg\alpha + 1}{tg^3\alpha}) =$$

$$\frac{tg\alpha \cancel{(tg^2\alpha - tg\alpha + 1)}}{1} \cdot \frac{tg^3\alpha}{\cancel{tg^2\alpha - tg\alpha + 1}} = tg\alpha \cdot tg^3\alpha = \boxed{tg^4\alpha}$$

Naravno, uslovi zadatka su da (pošto u imeniocu nesme da bude nula):

$$tg\alpha \neq 0 \quad \text{ i } \quad ctg\alpha \neq 0$$