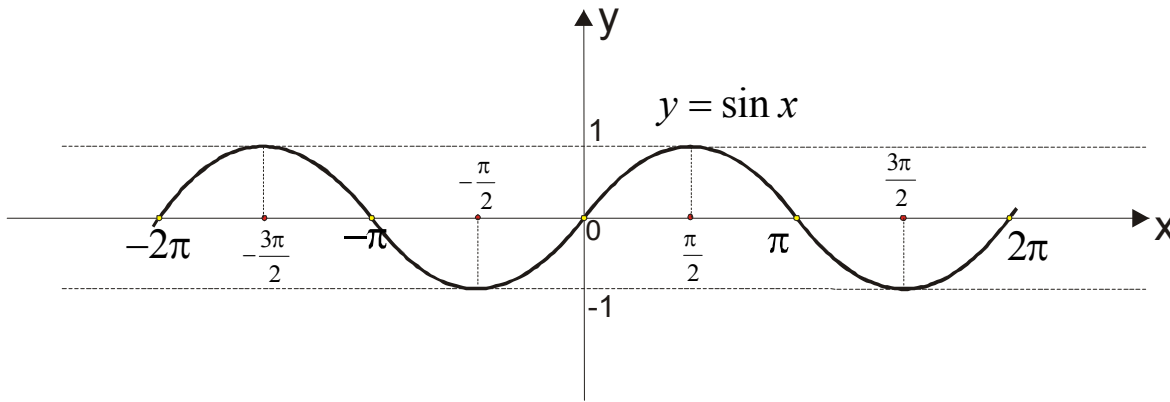


Trigonometric functions-graphics (part I)

$$y = a \sin(bx + c)$$

Recall the basic graphics $y = \sin x$ and its properties.



Features:

- Function defined for all x , that is : $x \in (-\infty, \infty)$
- Set the value function is an interval $[-1, 1]$, that is, a function is limited $-1 \leq \sin x \leq 1$
- $\sin x$ is a periodic function with the main period 2π
- **Zero function** (where the graph cuts x axis) are $x = 0, x = \pm\pi, x = \pm 2\pi \dots$ or it can be written, taking into account the periodicity $x = k\pi$ ($k = 0, \pm 1, \pm 2, \dots$)
- **Maximum value** functions are $-\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \dots$ we can write: $x = \frac{\pi}{2} + 2k\pi$ $k \in Z$
- **The minimum value** function has in $-\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \dots$ we can write: $x = -\frac{\pi}{2} + 2k\pi$ $k \in Z$
- $\sin x$ **increases** in intervals $[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi]$, $k \in Z$
- $\sin x$ **decreases** in the intervals $[\frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi]$, $k \in Z$
- Function is **positive**, $\sin x > 0$ for $x \in (2k\pi, (2k+1)\pi)$ $k \in Z$
- Function is **negative**, $\sin x < 0$ for $x \in ((2k-1)\pi, 2k\pi)$ $k \in Z$
- The graph is called a **sinusoid**

Trigonometric function $y = a \sin(bx + c)$ we will learn to draw in two ways.

The *first method* consists in the fact that we start from the initial graphics $y = \sin x$ and depending on the numbers a , b and c we moving graphics (we will learn how) and the *second way* is to directly test points (zero function, max, min ...) but for it we need to know to solve trigonometric equations.

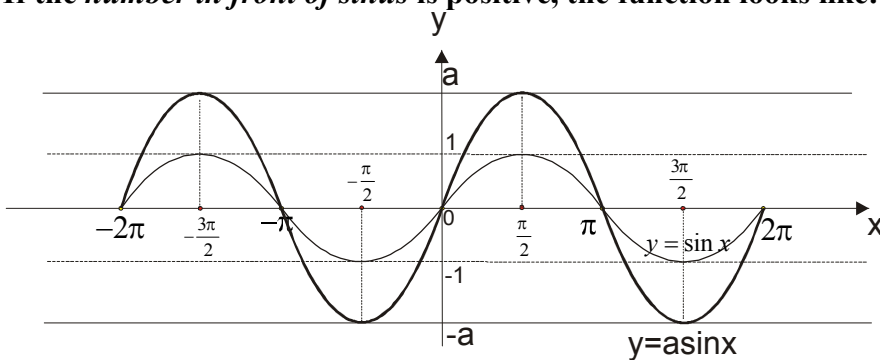
First, note and write down the numbers a , b and c .

$$y = a \sin x$$

Number a , which is in front of the sinuses is called **the amplitude** and it is the maximum distance point graphics from x-axis.

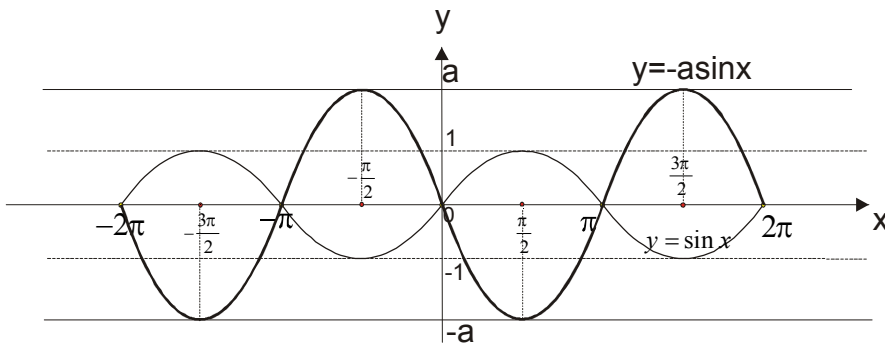
For the function $y = \sin x$ is the number $a = 1$ and we see that the graphic is just limited by -1 and 1

If the number in front of sinus is positive, the function looks like:



Therefore, the zero remains in place, while the max and min "extend" to the point a and $-a$.

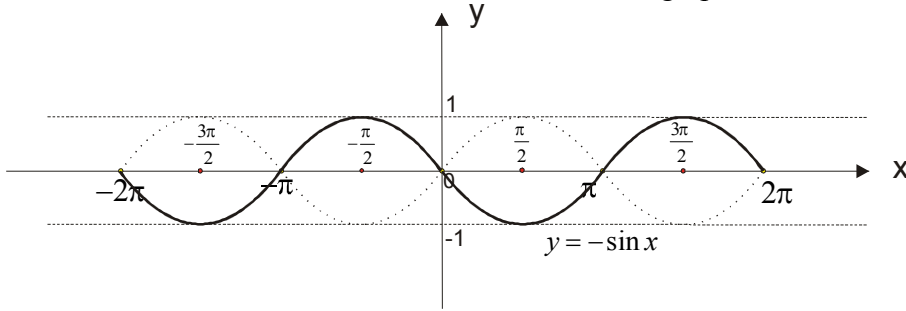
If the number in front of sinus is negative, the function looks like:



Watch out, here is the graph turns!

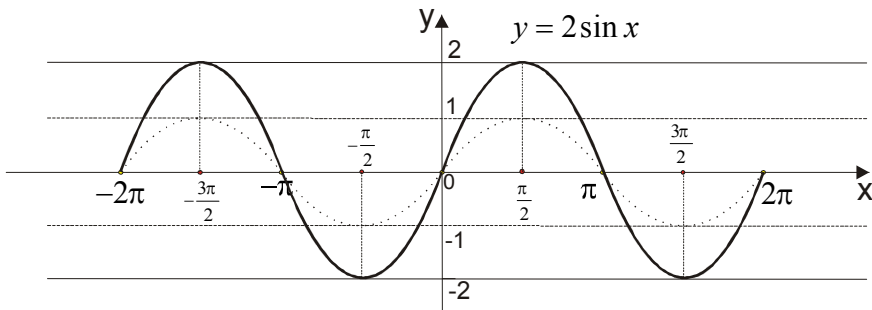
Example 1. Draw a graph $y = -\sin x$

Here we have $a = -1$. This indicates that the initial graph is turns.



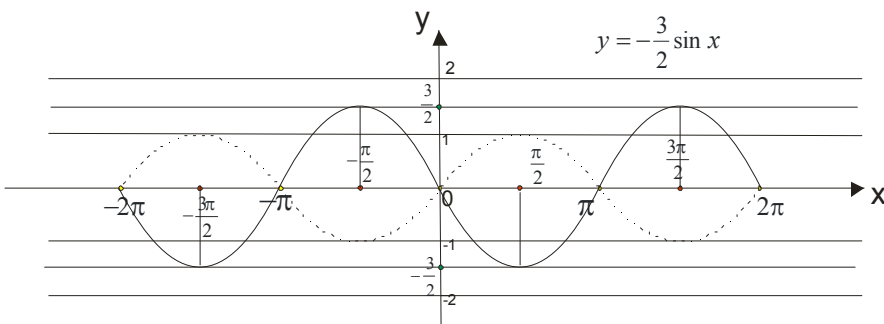
Example 2. Draw a graph $y = 2\sin x$

Now is $a = 2$. This means that the function of the y-axis goes from -2 to 2 and that the graph does not turn.



Example 3. Draw a graph $y = -\frac{3}{2}\sin x$

We have that $a = -\frac{3}{2}$.



$$y = \sin bx$$

Periodicity functions $y = a \sin(bx + c)$ directly follows from the periodicity of $y = \sin x$.

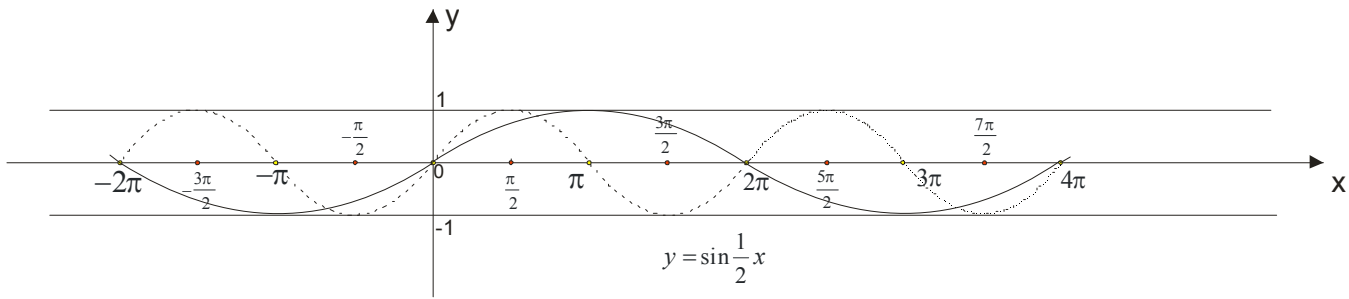
The main period of $y = a \sin(bx + c)$ is calculated by the formula $T = \frac{2\pi}{b}$.

The number b is called the *frequency*, and shows how the whole wave is on the interval $[0, 2\pi]$

Hence, our referred is to observe the number b , and insert it into $T = \frac{2\pi}{b}$ to get the base period.

Example 4. Draw a graph $y = \sin \frac{1}{2}x$

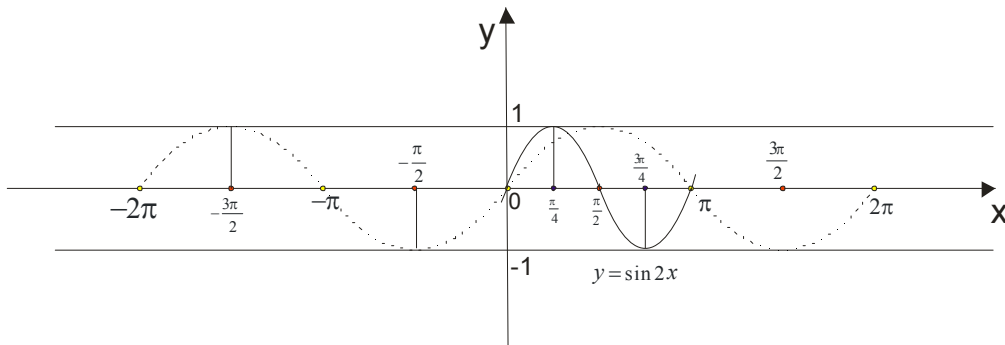
We notice that here $b = \frac{1}{2}$. Then $T = \frac{2\pi}{b} \rightarrow T = \frac{2\pi}{\frac{1}{2}} \rightarrow T = 4\pi$



Initial function $y = \sin x$ is here given intermittent. What happened to her? She “stretched” because $T = 4\pi$.

Example 5. Draw a graph $y = \sin 2x$

As is $b = 2$ then it the main period will be $T = \frac{2\pi}{b} \rightarrow T = \frac{2\pi}{2} \rightarrow T = \pi$



$$y = \sin(x + c)$$

or

$$y = \sin(bx + c)$$

(Number c is called the *initial stage*)

Again, of course, first read from the given function values for b and c . Then determine the value for $\frac{c}{b}$.

$y = \sin(bx + c)$ is obtained by moving graphics $y = \sin bx$ along the x axis and (watch this):

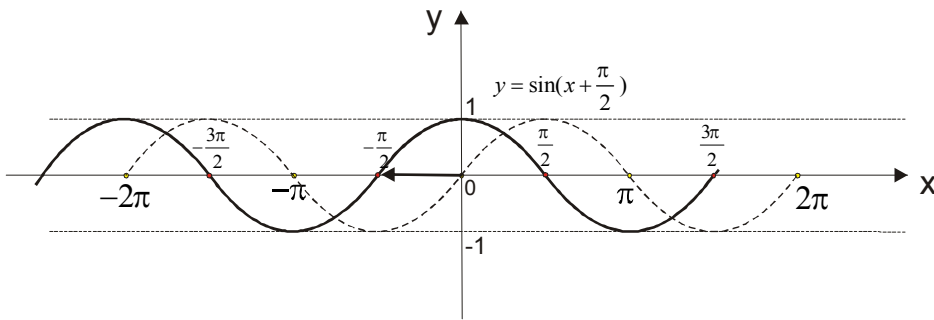
i) in a positive direction (right) if the value $\frac{c}{b}$ is negative

ii) in a negative direction (left) if the value of $\frac{c}{b}$ is positive

Example 6. Draw a graph $y = \sin(x + \frac{\pi}{2})$

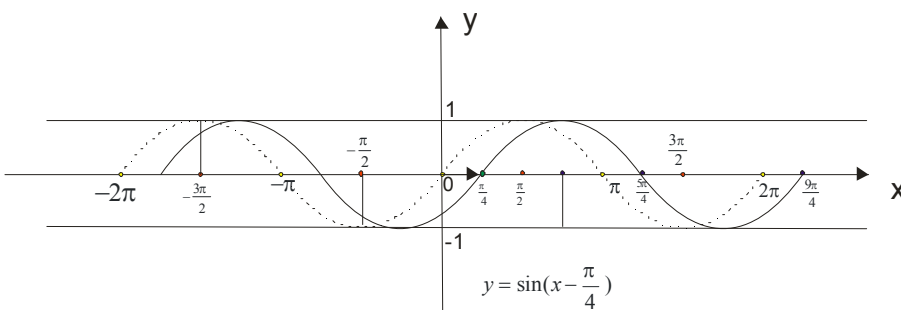
Here are $a=1, b=1, c = \frac{\pi}{2}$. Value of expression $\frac{c}{b}$ is $\frac{c}{b} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$. What does this mean?

Since the value of this expression is positive, the initial graph of $y = \sin x$ move for $\frac{\pi}{2}$ in left.



Example 7. Draw a graph $y = \sin(x - \frac{\pi}{4})$

$a=1, b=1, c = -\frac{\pi}{4} \rightarrow \frac{c}{b} = -\frac{\pi}{4}$ Move $y = \sin x$ for $\frac{\pi}{4}$ right.

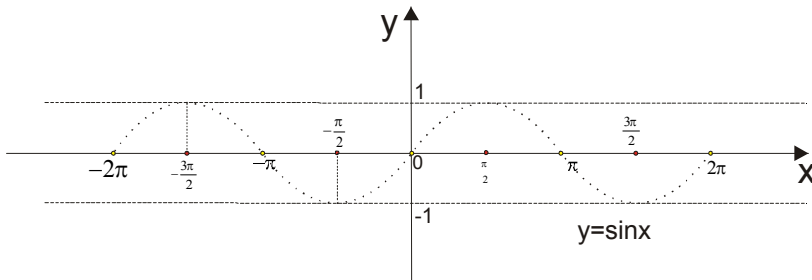


Example 8. Draw a graph $y = \sin(2x - \frac{\pi}{2})$

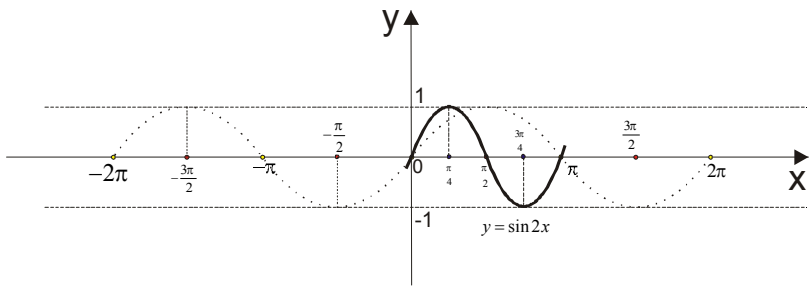
Here are $a=1$, $b=2$ and $c = -\frac{\pi}{2}$

$$T = \frac{2\pi}{b} \rightarrow T = \frac{2\pi}{2} \rightarrow \boxed{T = \pi} \quad \text{and} \quad \frac{c}{b} = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{4}$$

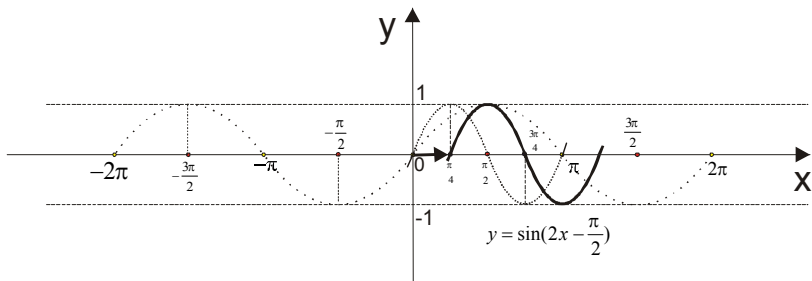
We have to draw **three** graphics : $y = \sin x$ (**slika 1.**) then $y = \sin 2x$ (**slika 2.**) and $y = \sin(2x - \frac{\pi}{2})$ (**slika 3.**)



slika 1.



slika 2.



slika 3.

On picture 2. we see that the graph of sine function "assembled" for period $T = \pi$.

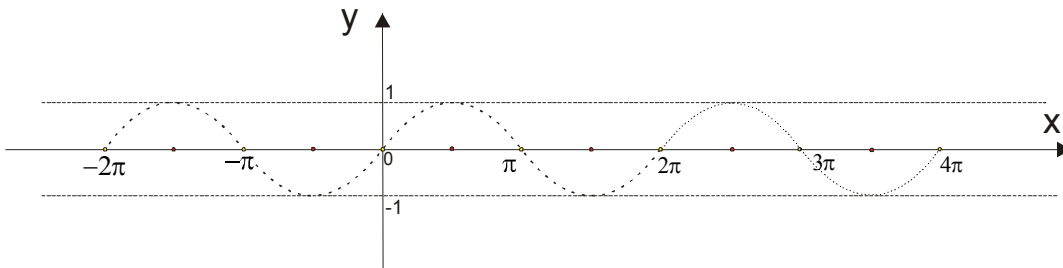
On picture 3. has been made moving graphic $y = \sin 2x$ for $\frac{\pi}{4}$ in right .

Your professor will probably ask you to apply all three graphics on one picture ... We intentionally draw three pictures for better understand...

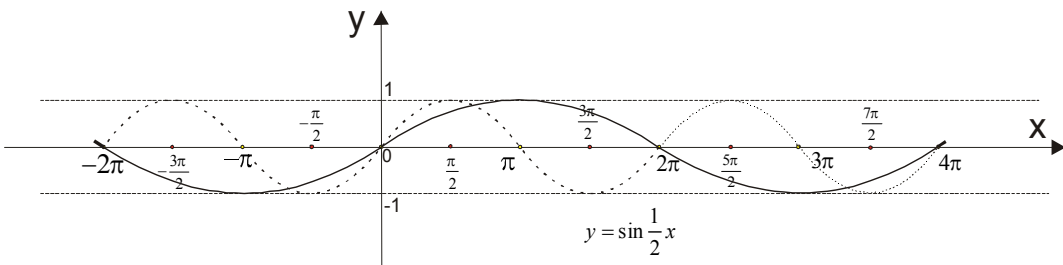
Example 9. Draw a graph $y = \sin\left(\frac{1}{2}x + \frac{\pi}{4}\right)$

$$a = 1, b = \frac{1}{2}, c = \frac{\pi}{4}$$

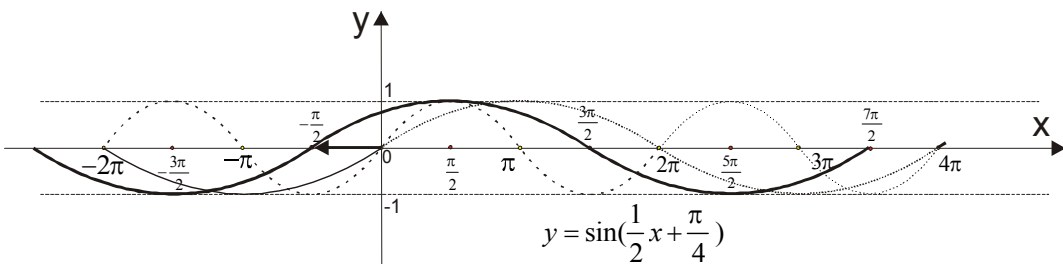
$$T = \frac{2\pi}{b} \rightarrow T = \frac{2\pi}{\frac{1}{2}} \rightarrow \boxed{T = 4\pi} \quad \text{and} \quad \frac{c}{b} = \frac{\frac{\pi}{4}}{\frac{1}{2}} = \frac{2\pi}{4} = \frac{\pi}{2}$$



slika 1.



slika 2.



slika 3.

Now we have the knowledge to draw the whole graph $y = a \sin(bx + c)$. See next file:

Trigonometric functions-graphics (part II)