Arithmetic progression (**arithmetic sequence)**

Head from the following two examples:

Example 1: 3,5,7,9,11,...

Example 2: 55,50,45,40,...

It is not difficult to conclude that in the first example the next few following members will be 13,15,17, because each member increases for two.

In example 2, a few next following members will be 35,30,25, ... each of them decreases by 5.

As we see, progression may be increasing or decreasing.

Arithmetic progression or **arithmetic sequence** is a sequence of numbers such that the difference of any two successive members of the sequence is a constant.

It is very important from which number arithmetic progression starts, and he is called **the first (**initial) term and marked with a_1.

For example 1, initial term is $a_1 = 3$

For example 2, initial term is $a_1 = 55$

The common difference of successive members is d (difference).

$d = a_2 - a_1 = a_3 - a_2 = ... = a_n - a_{n-1}$

For example 1, $d = 2$ (increas)

For example 2, $d = -5$ (decreas)

The *n*- th term of the sequence is given by:

$$a_n = a_1 + (n-1)d$$

The sum of the components of an arithmetic progression is called an **arithmetic series.**

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right] \quad \text{or} \quad S_n = \frac{n(a_1 + a_n)}{2}$$

For each arithmetical progression still applies: (arithmetic middle)

$$a_n = \frac{a_{n-1} + a_{n+1}}{2} \quad \text{or} \quad a_n = \frac{a_{n-j} + a_{n+j}}{2} \qquad j = 2,...,n-1$$

EXAMPLES:

1) The fifth member of arithmetic progression is 19 and the tenth member is 39 .Determine that arithmetic progression .

Solution:

$a_5 = 19$
$a_{10} = 39$ we will use formula: $a_n = a_1 + (n-1)d$

$a_n = a_1 + (n-1)d$ for $n = 5 \Rightarrow a_5 = a_1 + 4d = 19$
for $n = 10 \Rightarrow a_{10} = a_1 + 9d = 39$

Next, we form system of equations:

$$a_1 + 4d = 19 / \cdot (-1)$$
$$a_1 + 9d = 39$$

$$-a_1 - 4d = -19$$
$$+ \ a_1 + 9d = 39$$

$$5d = 20$$
$d = 4 \rightarrow$ back in one of the equation

$$a_1 + 4d = 19$$
$$a_1 + 16 = 19$$
$$a_1 = 3$$

So: arithmetic progression is

$$3, 7, 11, 15, 19, \ldots$$

The general member will be:

(in this example is not required)

$$a_n = a_1 + (n-1)d$$
$$a_n = 3 + (n-1) \cdot 4$$
$$a_n = 4n - 1$$

2. Find a_1 and **d** in arithmetic progression if : $a_2+a_5-a_3=10$ and $a_2+a_9=17$

Solution:

$$a_n=a_1+(n-1)d \rightarrow \begin{array}{l} a_2=a_1+d \\ a_5=a_1+4d \\ a_3=a_1+2d \\ a_9=a_1+8d \end{array}$$

Replace this in $a_2+a_5-a_3=10$ and $a_2+a_9=17$

$$(a_1+d)+(a_1+4d)-(a_1+2d)=10$$
$$(a_1+d)+(a_1+8d)=17$$

$$a_1+d+a_1+4d-a_1-2d=10$$
$$a_1+d+a_1+8d=17$$

$$a_1+3d=10 \rightarrow$$ multiply this equation with (-2)
$$2a_1+9d=17$$

$$3d=-3$$
$$d=-1$$
$$a_1+3d=10$$
$$a_1-3=10$$
$$a_1=13$$

So, this arithmetic progression is decreasing:

$$13,12,11,10,9,8,7,\dots$$

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3. Find arithmetic progression if : $5a_1+10a_5=0$ and $S_4=14$

Solution:

$$a_n=a_1+(n-1)d$$
$$a_5=a_1+4d$$
$$5a_1+10(a_1+4d)=0$$
$$5a_1+10a_1+40d=0$$
$$15a_1+40d=0$$
$$3a_1+8d=0$$

$$S_4=14$$
$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$
$$S_4=\frac{4}{2}\left[2a_1+(4-1)d\right]$$
$$14=2\left[2a_1+3d\right]$$
$$2a_1+3d=7$$

Now , from these two equations we make system:

$$3a_1 + 8d = 0 \,/\cdot 2$$
$$2a_1 + 3d = 7 \,/\cdot(-3)$$

$$6a_1 + 16d = 0$$
$$-6a_1 - 9d = -21$$

$$7d = -21$$
$$d = -3$$
$$3a_1 + 8d = 0 \Rightarrow 3a_1 - 24 = 0$$
$$3a_1 = 24$$
$$a_1 = 8$$

Arithmetic progression is: 8,5,2,-1,-4,…

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4. Determine the tenth member of arithmetical progression if:

$$a_1 = 2$$
$$d = 5$$
$$S_n = 245$$

Solution:

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$
$$245 = \frac{n}{2}\left[2\cdot 2 + (n-1)\cdot 5\right]$$
$$245 = \frac{n}{2}\left[4 + 5n - 5\right]$$
$$490 = n\left[5n - 1\right]$$
$$490 = 5n^2 - n$$
$$5n^2 - n - 490 = 0$$

We have received a square equation "by n".

$$a = 5, b = -1, c = -490$$

$$n_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$n_{1,2} = \frac{1 \pm 99}{10}$$
$$n_1 = 10, n_2 = -\frac{98}{10} \to impossible$$

So : $n = 10$ is the only solution

$$a_n = a_1 + (n-1)d$$
$$a_{10} = 2 + (10-1)\cdot 5$$
$$a_{10} = 2 + 45$$
$$a_{10} = 47$$

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5. The sum of the first three members of arithmetical progression is 36 and the sum of the squares of first three members is 482. Find that progression.

Solution:

To “place” the problem:

$$\begin{array}{l} a_1 + a_2 + a_3 = 36 \\ a_1^2 + a_2^2 + a_3^2 = 482 \end{array}$$

we can use that :

$$a_n = a_1 + (n-1)d$$
$$a_2 = a_1 + d$$
$$a_3 = a_1 + 2d$$

$$a_1 + (a_1 + d) + (a_1 + 2d) = 36$$
$$a_1^2 + (a_1 + d)^2 + (a_1 + 2d)^2 = 482$$

$$3a_1 + 3d = 36$$
$$a_1 + d = 12$$

From here we will express a_1 and replace in the second equation of system.

$$a_1 = 12 - d$$

$$(12-d)^2 + (12-d+d)^2 + (12-d+2d)^2 = 482$$
$$(12-d)^2 + 12^2 + (12+d)^2 = 482$$
$$144 - 24d + d^2 + 144 + 144 + 24d + d^2 = 482$$
$$2d^2 + 432 = 482$$
$$2d^2 = 50$$
$$d^2 = 25$$
$$d = \pm\sqrt{25}$$
$$d = 5$$
$$a_1 = 12 - 5$$
$$a_1 = 7$$

OR

$$d = -5$$
$$a_1 = 12 + 5$$
$$a_1 = 17$$

So: there are two solutions:

7,12,17,22,27,… and 17,12,7,2,-3,…

6. Solve the equation: $3+7+11+...+x=210$

Solution:

$$3+7+11+...+x=210 \longrightarrow \begin{array}{l} a_1=3 \\ a_2=7 \\ a_n=x \\ S_n=210 \end{array}$$

$$\begin{array}{l} a_1=3 \\ d=4 \\ S_n=210 \\ \hline x=a_n=? \end{array}$$

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$210=\frac{n}{2}\left[2\cdot 3+(n-1)\cdot 4\right]$$

So: $$210=\frac{n}{2}\left[6+4n-4\right]$$

$$210=\frac{n}{2}\left[4n+2\right]$$

$$210=2n^2+n$$

$$2n^2+n-210=0$$

$$n_{1,2}=\frac{-1\pm 41}{4}$$

$$n_1=10$$

$$n_2=-\frac{42}{4}$$

From here is: $n=10$

$$x=a_{10}=a_1+9d=3+9\cdot 4=3+36=39$$

$$x=39$$

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7. Determine x so that the numbers log2, log(2^x -1), log(2^x +3) be successive members of arithmetical progression.

Solution:

We will use that $$a_n=\frac{a_{n-1}+a_{n+1}}{2} \longrightarrow a_2=\frac{a_1+a_3}{2}$$

$$\log 2, \log(2^x - 1), \log(2^x + 3)$$

$$\log(2^x - 1) = \frac{\log 2 + \log(2^x + 3)}{2}$$

$$2\log(2^x - 1) = \log 2 \cdot (2^x + 3)$$

$$\log(2^x - 1)^2 = \log 2 \cdot (2^x + 3)$$

$$(2^x - 1)^2 = 2 \cdot (2^x + 3) \text{.....} \textit{replacement} \text{...} 2^x = t$$

$$(t - 1)^2 = 2(t + 3)$$

$$t^2 - 2t + 1 = 2t + 6$$

$$t^2 - 4t - 5 = 0$$

$$t_{1,2} = \frac{4 \pm 6}{2}$$

$$t_1 = 5$$

$$t_2 = -1$$

Back in the replacement:

$2^x = 5$ or $2^x = -1$ ⟶ impossible

$x = \log_2 5$

solution