

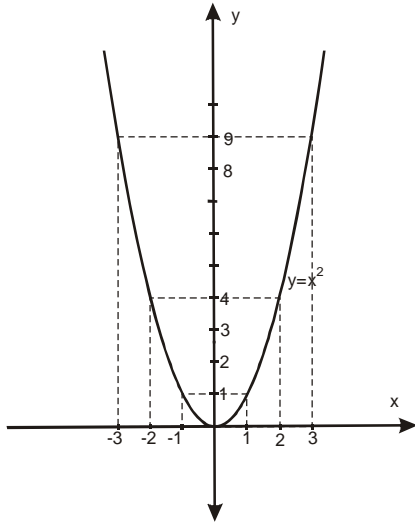
SQUARE FUNCTION $y = ax^2 + bx + c$

It is : $y = ax^2 + bx + c$ where is $x \in R$, $a \neq 0$ and a, b, c are real numbers.

Curve in the plane, which represents a graph function $y = ax^2 + bx + c$ is a **parable**.

First, we learn how to design graph of functions $y = x^2$. Create table for some value of variable x .

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



For $x = -3$

$$y = (-3)^2 = 9$$

For $x = 2$

$$y = 2^2 = 4$$

For $x = -2$

$$y = (-2)^2 = 4$$

For $x = 3$

$$y = 3^2 = 9$$

For $x = -1$

$$y = (-1)^2 = 1$$

For $x = 0$

$$y = 0^2 = 0$$

For $x = 1$

$$y = 1^2 = 1$$

Learn now graph $y = ax^2$

There are 2 situations: $a > 0$ and $a < 0$

For $a > 0$

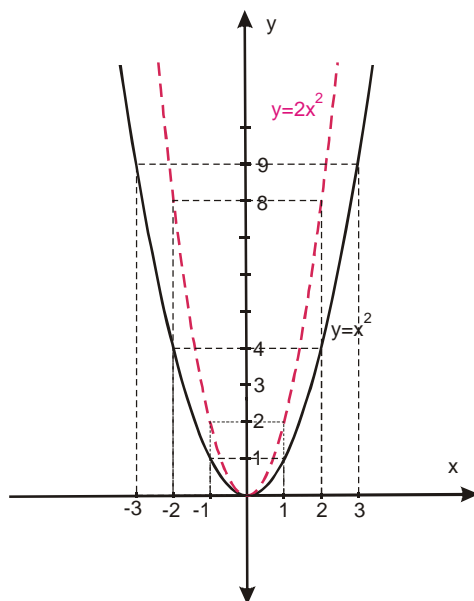
Here is a parable with "opening up". What happens if $a > 1$ and if $0 < a < 1$?

$a > 1$

In relation to the initial graph $y = x^2$, graph $y = ax^2 (a > 1)$ is **“more thin”**

Example: $y = 2x^2$

x	-3	-2	-1	0	1	2	3
y	18	8	2	0	2	8	18

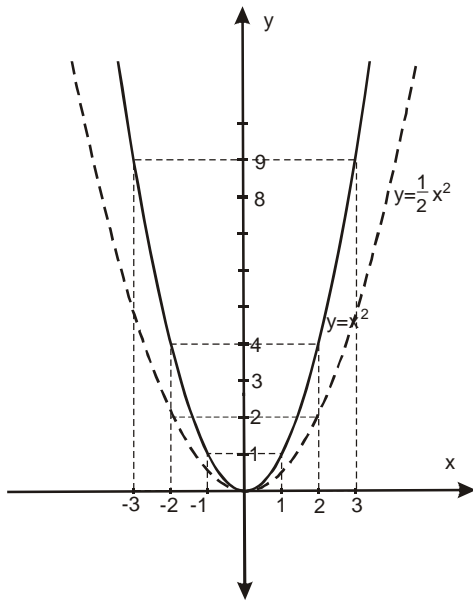


$0 < a < 1$

In relation to the initial graph $y = x^2$, graph $y = ax^2 (0 < a < 1)$ is **“more fat”**

Example: $y = \frac{1}{2}x^2$

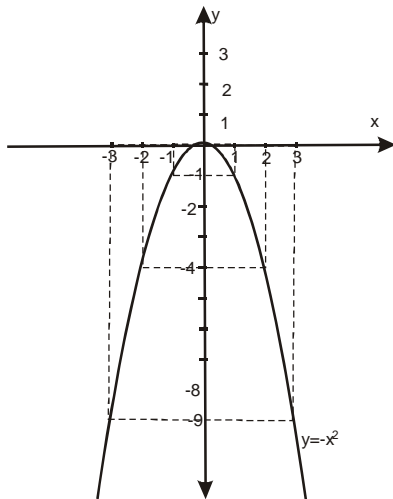
x	-3	-2	-1	0	1	2	3
y	$\frac{9}{2}$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2	$\frac{9}{2}$



For $a < 0$ here is a parabola with "the opening down."

Initial graph is $y = -x^2$.

x	-3	-2	-1	0	1	2	3
y	-9	-8	-2	0	-2	-8	-9

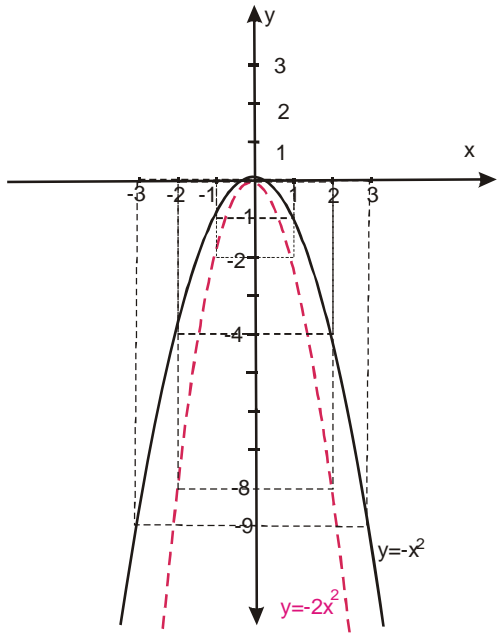


Again, we consider 2 situations for function $y = ax^2$: for $a < -1$ and $-1 < a < 0$

The following two examples will clarify us their "behavior":

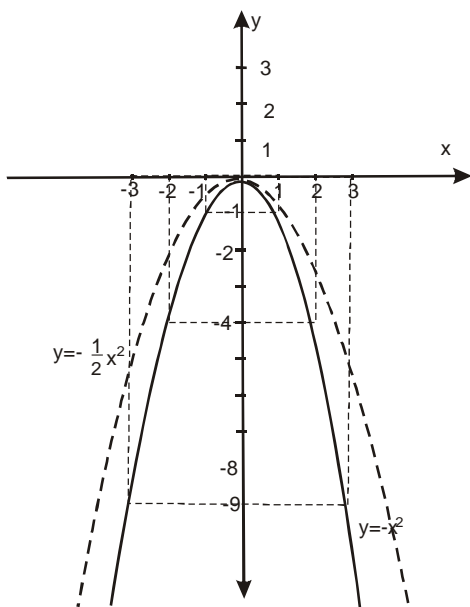
Example 1: $y = -2x^2$

x	-3	-2	-1	0	1	2	3
y	-18	-8	-2	0	-2	-8	-18



Example 2. : $y = -\frac{1}{2}x^2$

x	-3	-2	-1	0	1	2	3
y	$-\frac{9}{2}$	-2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-2	$-\frac{9}{2}$



How to draw a graph $y = ax^2 + \beta$?

→ first draft graph of function $y = ax^2$

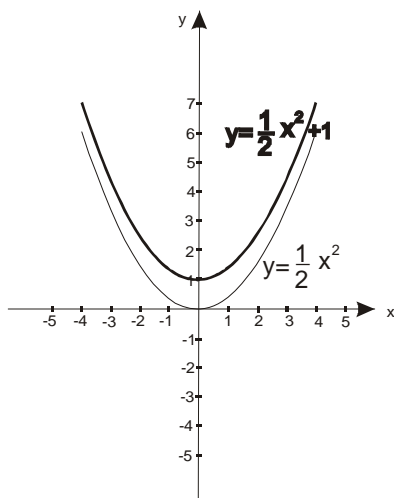
→ that graph then move along y-line for value of β :

1) If β is positive, move it in the positive direction along y- line

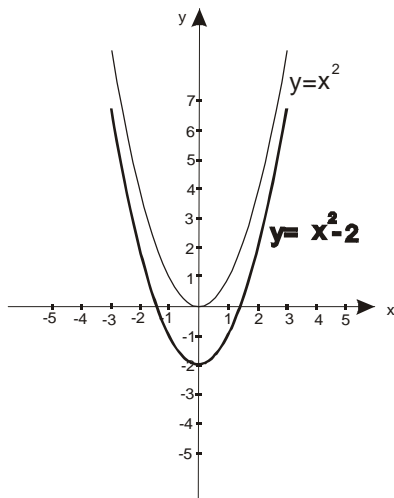
2) If β is negative, move it in the negative direction along y- line

Here are a few examples:

Example 1. $y = \frac{1}{2}x^2 + 1$ (here is $\beta = 1$, and we move “up”)



Example 2. $y = x^2 - 2$ (here is $\beta = -2$ and we move “down”)



How to draw a graph $y = (x - \alpha)^2$?

→ first draft graph of function $y = x^2$ (or $y = -x^2$)

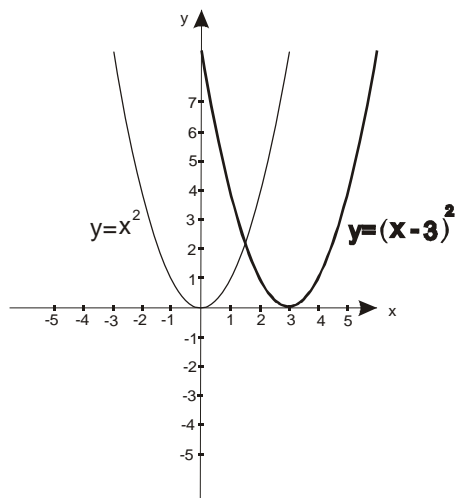
→ that graph then move along x-line :

1) If we have $x + \alpha$, move it in the negative direction along x- line (α is negative)

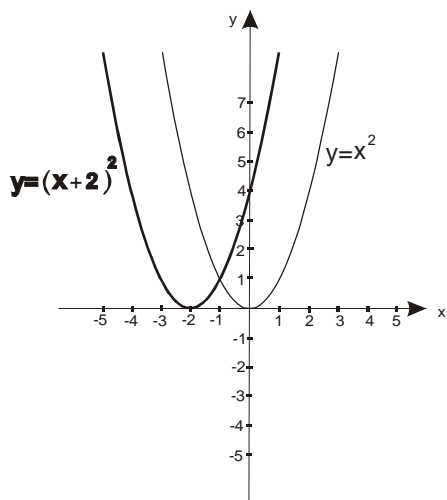
2) If we have $x - \alpha$, move it in the positive direction along x- line (α is positive)

Here are a few examples:

Example 1. $y = (x - 3)^2$ (here is $\alpha = 3$, and we move in positive direction)



Example 2. $y = (x + 2)^2$ (here is $\alpha = -2$, and we move in negative direction)



Now we have the knowledge to draft the graph of function $y = ax^2 + bx + c$

First, we have to use so-called “canonical form” : $y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$

Or, if we mark $\alpha = -\frac{b}{2a}$ and $\beta = \frac{4ac - b^2}{4a}$, we have : $y = a(x - \alpha)^2 + \beta$ and thih form we will use.

Important procedure

- i) **Put $y = ax^2 + bx + c$ in canonical form : $y = a(x - \alpha)^2 + \beta$**
- ii) **Draw graph of function $y = ax^2$**
- iii) **Process move along the x-line for α**
- iv) **Process move along the y-line for β**

Example: Draw a graph $y = x^2 - 6x + 5$

Solution:

- i) **Put $y = ax^2 + bx + c$ in canonical form : $y = a(x - \alpha)^2 + \beta$**

$$a = 1 \quad \alpha = -\frac{b}{2a} = -\frac{-6}{2 \cdot 1} = 3$$

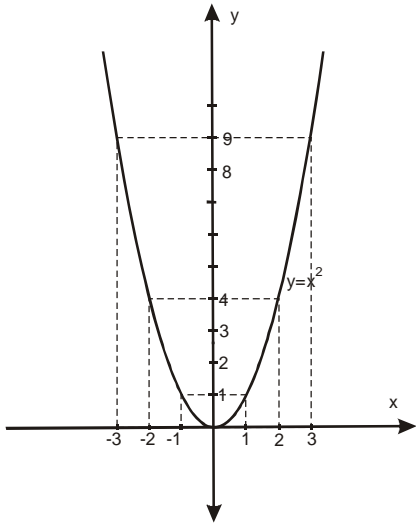
$$b = -6$$
$$c = 5 \quad \beta = \frac{4ac - b^2}{4a} = \frac{4 \cdot 1 \cdot 5 - (-6)^2}{4 \cdot 1} = \frac{20 - 36}{4} = \frac{-16}{4} = -4$$

$$y = a(x - \alpha)^2 + \beta$$

$$y = 1(x - 3)^2 + (-4)$$

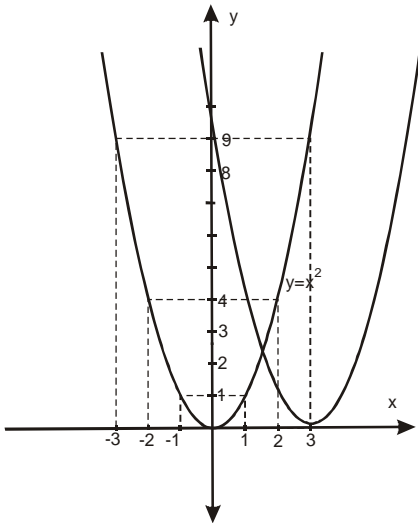
$$y = (x - 3)^2 - 4$$

ii) Draw graph of function $y = ax^2 \longrightarrow y = x^2$ Because ($a=1$)



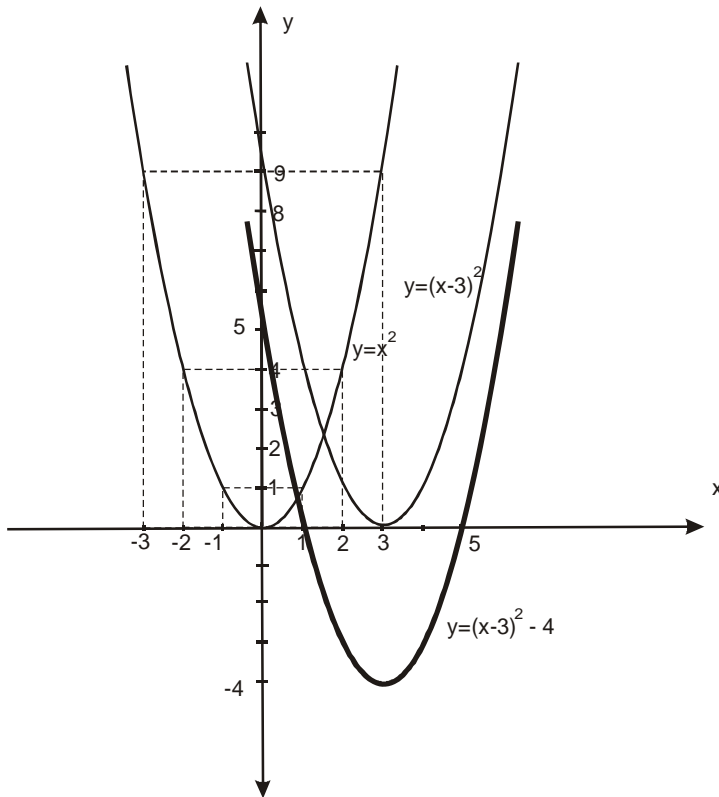
iii) Process move along the x-line for α

$$y = x^2 \longrightarrow y = (x-3)^2$$



iv) Process move along the y-line for β

$$y = (x-3)^2 \longrightarrow y = (x-3)^2 - 4$$



The whole process is little "complicated" and not even much accurate. Here's how to more quickly and accurately draw a graph $y = ax^2 + bx + c$ without reducing the canonical form and use other "movements":

Procedure:

1) First, determine a, b, c , and find $D = b^2 - 4ac$


2) $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$


$D > 0, x_1 \neq x_2$

$D = 0, x_1 = x_2$

$D < 0, No... x_1, x_2$

3) Depending on the number a , we have:

i) If $a > 0$ 

ii) If $a < 0$ 

4) Parable always cutting the y-line in the number c

5) $T(\alpha, \beta) \quad \alpha = -\frac{b}{2a}, \beta = -\frac{D}{4a}$

$T(\alpha, \beta)$ is max if $a < 0$

$T(\alpha, \beta)$ is min if $a > 0$

Example 1): $y = x^2 - 6x + 5$

Solution:

1)

$$a = 1 \quad D = b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot 5 = 36 - 20 = 16$$

$$b = -6$$

$$c = 5$$

2)

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{6 \pm 4}{2}$$

$$x_1 = 5$$

$$x_2 = 1$$

3)

$$a = 1 > 0 \Rightarrow \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array}$$

4)

y-line is cutting in $c=5$

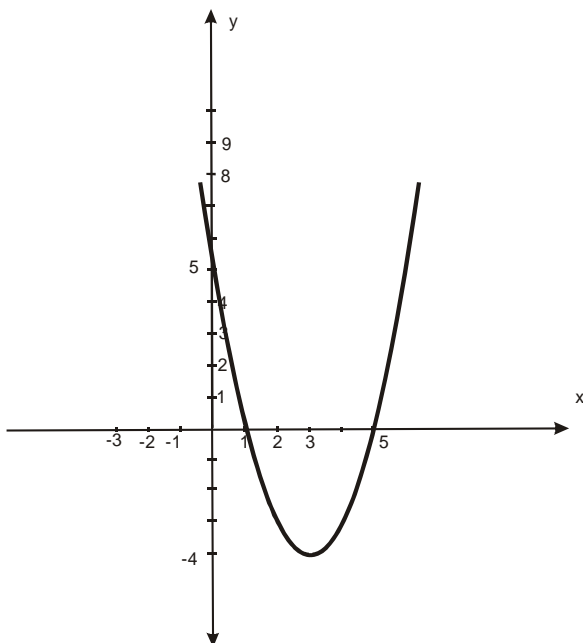
$$T(\alpha, \beta)$$

$$\alpha = -\frac{b}{2a} = \frac{-6}{2 \cdot 1} = 3$$

5)

$$\beta = -\frac{D}{4a} = -\frac{16}{4 \cdot 1} = -4$$

$$\underbrace{T(3, -4)}_{\text{min}}$$



Example 2) $y = -\frac{1}{2}x^2 + \frac{1}{2}x + 6$

Solution:

1) $a = -\frac{1}{2}$

$b = \frac{1}{2}$

$c = 6$

$$D = \left(\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) \cdot 6 = -\frac{1}{4} + 12 = 12\frac{1}{4} = \frac{49}{4}$$

2)

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{\frac{1}{2} \pm \frac{7}{2}}{2\left(-\frac{1}{2}\right)} = \frac{-\frac{1}{2} \pm \frac{7}{2}}{-1}$$

$$x_1 = -3$$

$$x_2 = 4$$

3)

$$a = -\frac{1}{2} < 0 \Rightarrow \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array}$$

4)

y-line is cutting in $c=6$

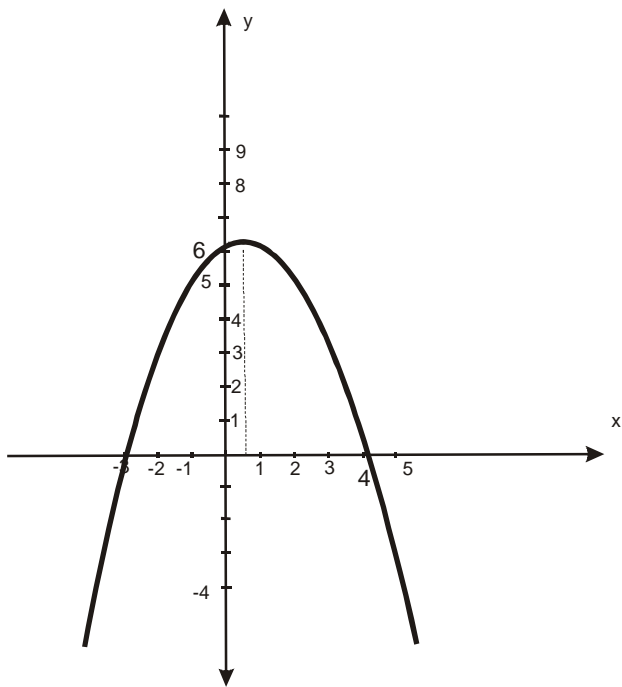
5)

$T(\alpha, \beta)$

$$\alpha = -\frac{b}{2a} = -\frac{\frac{1}{2}}{2\left(-\frac{1}{2}\right)} = \frac{1}{2}$$

$$\beta = -\frac{D}{4a} = -\frac{\frac{49}{4}}{4\left(-\frac{1}{2}\right)} = +\frac{49}{8} = 6\frac{1}{8}$$

$$T\left(\frac{1}{2}, 6\frac{1}{8}\right)$$



Example 3) $y = x^2 - 4|x| + 3$

Solution:

Because $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ here we have 2 graphics, one for $x \geq 0$, and one for $x < 0$

$$y = x^2 - 4|x| + 3$$

for $x \geq 0$

$$\underline{y = x^2 - 4x + 3}$$

1)

$$a = 1, b = -4, c = 3$$

$$D = b^2 - 4ac = 16 - 12 = 4$$

$$x_{1,2} = \frac{4 \pm 2}{2}$$

2)

$$x_1 = 3$$

$$x_2 = 1$$

3)

$$a = 1 > 0 \Rightarrow \text{parabola opening upwards}$$

4) y-line is cutting in 3

5)

$T(\alpha, \beta)$

$$\alpha = -\frac{b}{2a} = \frac{-4}{2 \cdot 1} = 2$$

$$\beta = -\frac{D}{4a} = -\frac{4}{4 \cdot 1} = -1$$

$T(2, -1)$

For $x < 0$

$y = x^2 + 4x + 3$

1)

$$a = 1, b = 4, c = 3$$

$$D = b^2 - 4ac = 16 - 12 = 4$$


2)

$$x_{1,2} = \frac{-4 \pm 2}{2}$$

$$x_1 = -1$$

$$x_2 = -3$$

3)

$a = 1 > \Rightarrow$ 

4)

y-line is cutting in 3

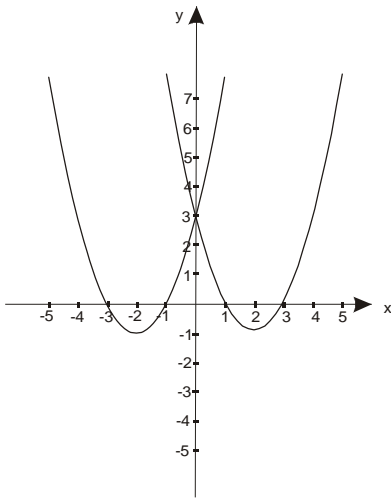
5)

$T(\alpha, \beta)$

$$\alpha = -\frac{b}{2a} = \frac{-4}{2 \cdot 1} = -2$$

$$\beta = -\frac{D}{4a} = -\frac{4}{4 \cdot 1} = -1$$

$T(-2, -1)$



First graph $y = x^2 - 4x + 3$ we need only for $x \geq 0$ and
 second graph $y = x^2 + 4x + 3$ for $x < 0$

final solution (FOR $y = x^2 - 4|x| + 3$) is:

