

Some equations that are scaled to the square equation

1) Type of equation $ax^4 + bx^2 + c = 0$

This type of equation we solve with replacement $x^2 = t$, get equation $at^2 + bt + c = 0$, and

find $t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; then we go back to replacement:

$$\begin{array}{l} x^2 = t_1 \quad \text{and} \quad x^2 = t_2 \\ x_{1,2} = \pm\sqrt{t_1} \quad \text{and} \quad x_{3,4} = \pm\sqrt{t_2} \end{array}$$

Example 1. Solve the equation: $x^4 - 4x^2 + 3 = 0$

Solution:

$$x^4 - 4x^2 + 3 = 0 \Rightarrow \text{replacement } x^2 = t$$

$$t^4 - 4t^2 + 3 = 0$$

$$a = 1$$

$$b = -4$$

$$c = 3$$

$$t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$$

$$t_{1,2} = \frac{4 \pm \sqrt{16 - 12}}{2} = \frac{4 \pm 2}{2}$$

$$t_1 = \frac{4 + 2}{2} = 3$$

$$t_2 = \frac{4 - 2}{2} = 1$$

Back in $x^2 = t$:

$$x^2 = t_1$$

$$x^2 = 3$$

$$x_{1,2} = \pm\sqrt{3}$$

$$x_1 = +\sqrt{3}$$

$$x_2 = -\sqrt{3}$$

$$x^2 = t_2$$

$$x^2 = 1$$

$$x_{3,4} = \pm\sqrt{1}$$

$$x_3 = +1$$

$$x_4 = -1$$

Example 2. Solve the equation: $(4x^2 - 5)^2 + (x^2 + 5)^2 = 2(8x^4 - 83)$

Solution:

$$(4x^2 - 5)^2 + (x^2 + 5)^2 = 2(8x^4 - 83)$$

$$16x^4 - 40x^2 + 25 + x^4 + 10x^2 + 25 = 16x^4 - 166$$

$$x^4 - 30x^2 + 50 + 166 = 0$$

$$x^4 - 30x^2 + 216 = 0 \rightarrow \text{replacement : } x^2 = t$$

$$t^2 + 30t + 216 = 0$$

$$a = 1$$

$$b = -30$$

$$c = 216$$

$$t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{30 \pm \sqrt{900 - 864}}{2}$$

$$t_{1,2} = \frac{30 \pm 6}{2}$$

$$t_1 = \frac{36}{2} = 18$$

$$t_2 = \frac{24}{2} = 12$$

Now:

$$x^2 = 18$$

$$x_{1,2} = \pm\sqrt{18}$$

$$x_{1,2} = \pm 3\sqrt{2}$$

$$x_1 = +3\sqrt{2}$$

$$x_2 = -3\sqrt{2}$$

$$x^2 = 12$$

$$x_{3,4} = \pm\sqrt{12}$$

$$x_{3,4} = \pm 2\sqrt{2}$$

$$x_3 = +2\sqrt{2}$$

$$x_4 = -2\sqrt{2}$$

Example 3. Solve the equation: $(x^2 - 2x)^2 - 2(x^2 - 2x) = 3$

Solution:

It is better to take the replacement: $x^2 - 2x = t$

$$(x^2 - 2x)^2 - 2(x^2 - 2x) = 3$$

$$t^2 - 2t = 3$$

$$t^2 - 2t - 3 = 0$$

$$a = 1$$

$$b = -2$$

$$c = -3$$

$$t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_{1,2} = \frac{2 \pm \sqrt{4 + 12}}{2} = \frac{2 \pm 4}{2}$$

$$t_1 = 3$$

$$t_2 = -1$$

Let's go back now to $x^2 - 2x = t$

$$x^2 - 2x = t_1$$

$$x^2 - 2x = 3$$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 2x = t_2$$

$$x^2 - 2x = -1$$

$$x^2 - 2x + 1 = 0$$

We have now two new square equation :

$$x^2 - 2x - 3 = 0 \longrightarrow \begin{matrix} a = 1 \\ b = -2 \\ c = -3 \end{matrix} \longrightarrow \begin{matrix} x_{1,2} = \frac{2 \pm \sqrt{4+12}}{2} \\ x_{1,2} = \frac{2 \pm 4}{2} \\ x_1 = 3 \\ x_2 = -1 \end{matrix}$$

$$x^2 - 2x + 1 = 0 \longrightarrow \begin{matrix} a = 1 \\ b = -2 \\ c = 1 \end{matrix} \longrightarrow \begin{matrix} x_{3,4} = \frac{2 \pm \sqrt{4-4}}{2} \\ x_{3,4} = \frac{2 \pm 0}{2} \\ x_3 = 1 \\ x_4 = 1 \end{matrix}$$

Therefore, solutions are: $\{3, -1, 1, 1\}$

Example 4: Solve the equation: $x(x+1)(x+2)(x+3) = 0,5625$

Solution: If we multiply all, then we have the problem!

Try to multiply the first two, and the other two, to see what will drop ..

$$(x^2 + x)(x^2 + 3x + 2x + 6) = 0,5625$$

$$(x^2 + x)(x^2 + 5x + 6) = 0,5625 \rightarrow \text{Not good!}$$

Try to multiply the first and fourth, and second and third

$$x(x+1)(x+2)(x+3) = 0,5625$$

$$(x^2 + 3x)(x^2 + 2x + 1x + 2) = 0,5625$$

$$(x^2 + 3x)(x^2 + 3x + 2) = 0,5625$$

Well, this is the better \Rightarrow replacement: $x^2 + 3x = t$

$$t \cdot (t + 2) = 0,5625$$

$$t^2 + 2t - 0,5625 = 0$$

$$t_{1,2} = \frac{-2 \pm \sqrt{4 + 2,25}}{2} = \frac{-2 \pm 2,5}{2}$$

$$t_1 = +0,25$$

$$t_2 = -2,25$$

$$x^2 + 3x = t_1$$

$$x^2 + 3x = +0,25$$

$$x^2 + 3x - 0,25 = 0$$

$$x_{1,2} = \frac{-3 \pm \sqrt{9 + 1}}{2}$$

$$x_{1,2} = \frac{-3 \pm \sqrt{10}}{2}$$

$$x_1 = \frac{-3 + \sqrt{10}}{2}$$

$$x_2 = \frac{-3 - \sqrt{10}}{2}$$

$$x^2 + 3x = t_2$$

$$x^2 + 3x = -2,25$$

$$x^2 + 3x + 2,25 = 0$$

$$x_{3,4} = \frac{-3 \pm \sqrt{9 - 9}}{2}$$

$$x_{3,4} = \frac{-3 \pm \sqrt{0}}{2}$$

$$x_3 = x_4 = -\frac{3}{2}$$

Example 5. Solve the equation: $\frac{x^2 + x - 5}{x} + \frac{3x}{x^2 + x - 5} + 4 = 0$

Solution:

$$\frac{x^2 + x - 5}{x} + \frac{3x}{x^2 + x - 5} + 4 = 0$$

$$\frac{x^2 + x - 5}{x} + 3 \cdot \frac{x}{x^2 + x - 5} + 4 = 0$$

It is useful to take replacement $\frac{x^2 + x - 5}{x} = t$ because then $\frac{x}{x^2 + x - 5} = \frac{1}{t}$

$$t + 3 \cdot \frac{1}{t} + 4 = 0 / \cdot t$$

$$t^2 + 3 + 4t = 0$$

$$t^2 + 4t + 3 = 0$$

$$t_{1,2} = \frac{-4 \pm \sqrt{16 - 12}}{2} = \frac{-4 \pm 2}{2}$$

$$t_1 = -1$$

$$t_2 = -3$$

Back in the replacement $\frac{x^2 + x - 5}{x} = t$

$$\frac{x^2 + x - 5}{x} = -1$$

$$x^2 + x - 5 = -x$$

$$x^2 + x - 5 + x = 0$$

$$x^2 + 2x - 5 = 0$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4 + 20}}{2}$$

$$x_{1,2} = \frac{-2 \pm \sqrt{24}}{2}$$

$$x_{1,2} = \frac{-2 \pm 2\sqrt{6}}{2}$$

$$x_{1,2} = \frac{2(-1 \pm \sqrt{6})}{2}$$

$$x_1 = -1 + \sqrt{6}$$

$$x_2 = -1 - \sqrt{6}$$

$\{-1 + \sqrt{6}; -1 - \sqrt{6}, 1, -5\}$ are solutions

or

$$\frac{x^2 + x - 5}{x} = -3$$

$$x^2 + x - 5 = -3x$$

$$x^2 + x - 5 + 3x = 0$$

$$x^2 + 4x - 5 = 0$$

$$x_{3,4} = \frac{-4 \pm \sqrt{16 + 20}}{2}$$

$$x_{3,4} = \frac{-4 \pm 6}{2}$$

$$x_3 = 1$$

$$x_4 = -5$$

2) Binomial equations $Ax^n \pm B = 0$

These are the types of equations: $Ax^n \pm B = 0$ where $A > 0$ and $B > 0$.

First, we try to separate the equation, using well-known formula, in factors, and use:

$$M \cdot N = 0 \Leftrightarrow M = 0 \vee N = 0$$

This equation can be always solved by the replacement $x = y^n \sqrt[n]{\frac{B}{A}}$, when we get shape $y^n \pm 1 = 0$

Example 1. Solve the equation: $8x^3 - 27 = 0$

Solution:

$$8x^3 - 27 = 0$$

$$(2x)^3 - 3^3 = 0$$

Watch out: it is wrong $8x^3 = 27$ because we “lost” solutions!

$$x^3 = \frac{27}{8}$$

$$8x^3 - 27 = 0$$

$$(2x)^3 - 3^3 = 0$$

Use formula:

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

$$(2x - 3)((2x)^2 + 2x \cdot 3 + 3^2) = 0$$

$$(2x - 3)(4x^2 + 6x + 9) = 0 \Rightarrow$$

$$2x - 3 = 0$$

$$4x^2 + 6x + 9 = 0$$

$$2x = 3$$

or

$$x_{2,3} = \frac{-6 \pm \sqrt{36 - 144}}{8}$$

$$x_1 = \frac{3}{2}$$

$$x_{2,3} = \frac{-6 \pm \sqrt{-108}}{8} = \frac{-6 \pm 6\sqrt{3}i}{8}$$

$$x_2 = \frac{-6 + 6\sqrt{3}i}{8}$$

$$x_3 = \frac{-6 - 6\sqrt{3}i}{8}$$

$$x_2 = \frac{-6 + 6\sqrt{3}i}{8} = \frac{2(-3 + 3\sqrt{3}i)}{8} = \frac{-3 + 3\sqrt{3}i}{4}$$

$$x_3 = \frac{-6 - 6\sqrt{3}i}{8} = \frac{2(-3 - 3\sqrt{3}i)}{8} = \frac{-3 - 3\sqrt{3}i}{4}$$

Take heed: $\sqrt{-108} = \sqrt{108} \cdot \sqrt{-1} = \sqrt{36 \cdot 3} \cdot i = 6\sqrt{3}i !!!$

Example 2. Solve the equation: $x^6 - 729 = 0$

Solution:

$$x^6 - 729 = 0$$

$$x^6 - 3^6 = 0$$

$$(x^3)^2 - (3^3)^2 = 0 \rightarrow \text{formula } A^2 - B^2 = (A - B)(A + B)$$

$$(x^3 - 3^3)(x^3 + 3^3) = 0$$

$$(x-3)(x^2 + 3x + 9)(x+3)(x^2 - 3x + 9) = 0$$

$$x-3=0 \quad \vee \quad x^2 + 3x + 9 = 0 \quad \vee \quad x+3=0 \quad \vee \quad x^2 - 3x + 9 = 0$$

$$x_1 = 3$$

$$x_{2,3} = \frac{-3 \pm \sqrt{9-36}}{2}$$

$$x_{2,3} = \frac{-3 \pm \sqrt{-27}}{2} = \frac{-3 \pm 3\sqrt{3}i}{2}$$

$$x_2 = \frac{-3 + 3\sqrt{3}i}{2} \quad x_3 = \frac{-3 - 3\sqrt{3}i}{2}$$

$$x_4 = -3 \quad x_{5,6} = \frac{3 \pm \sqrt{9-36}}{2}$$

$$x_5 = \frac{3 + 3\sqrt{3}i}{2} \quad x_6 = \frac{3 - 3\sqrt{3}i}{2}$$

Example 3. Solve the equation: $5x^3 + 2 = 0$

Solution:

$$\text{replacement : } x = y^{\frac{1}{n}} \sqrt[n]{\frac{B}{A}} \longrightarrow A=5, B=2, n=3$$

$$x = y^{\frac{1}{3}} \sqrt[3]{\frac{2}{5}}$$

$$5 \cdot \left(y^{\frac{1}{3}} \sqrt[3]{\frac{2}{5}} \right)^3 + 2 = 0$$

$$5 \cdot y^3 \cdot \frac{2}{5} + 2 = 0$$

$$y^3 \cdot 2 + 2 = 0$$

$$2 \cdot (y^3 + 1) = 0 \Rightarrow y^3 + 1 = 0$$

$$(y+1)(y^2 - y - 1) = 0$$

$$y+1=0$$

$$y_1 = -1$$

or

$$y^2 - y - 1 = 0$$

$$y_{2,3} = \frac{1 \pm \sqrt{1-4}}{2}$$

$$y_{2,3} = \frac{1 \pm i\sqrt{3}}{2}$$

$$y_2 = \frac{1 + i\sqrt{3}}{2}$$

$$y_3 = \frac{1 - i\sqrt{3}}{2}$$

Back in the replacement $x = y\sqrt[3]{\frac{2}{5}}$:

$$x = y\sqrt[3]{\frac{2}{5}}$$

$$x_1 = -1 \cdot \sqrt[3]{\frac{2}{5}} = -\sqrt[3]{\frac{2}{5}}$$

$$x_2 = \frac{1+i\sqrt{3}}{2} \cdot \sqrt[3]{\frac{2}{5}} \quad \text{and} \quad x_3 = \frac{1-i\sqrt{3}}{2} \cdot \sqrt[3]{\frac{2}{5}}$$

Example 4. Solve the equation: $11x^4 - 17 = 0$

Solution:

$$\text{replacement : } x = y\sqrt[n]{\frac{B}{A}} \longrightarrow n=4, \quad B=17, \quad A=11 \Rightarrow x = y\sqrt[4]{\frac{17}{11}}$$

$$11 \cdot \left(y\sqrt[4]{\frac{17}{11}} \right)^4 - 17 = 0$$

$$11 \cdot y^4 \frac{17}{11} - 17 = 0$$

$$11 \cdot y^4 - 17 = 0 \Rightarrow 17(y^4 - 1) = 0 \Rightarrow y^4 - 1 = 0$$

$$(y^2 - 1)(y^2 + 1) = 0$$

$$(y-1)(y+1)(y^2 + 1) = 0$$

$$y-1=0 \quad \vee \quad y+1=0 \quad \vee \quad y^2+1=0$$

$$y_1=1 \quad \vee \quad y_2=-1 \quad \vee \quad y^2=-1$$

$$y_{3,4} = \pm\sqrt{-1} = \pm i$$

$$y_3 = +i$$

$$y_4 = -i$$

$$\text{Back in } x = y\sqrt[4]{\frac{17}{11}}$$

$$x_1 = 1 \cdot \sqrt[4]{\frac{17}{11}} = \sqrt[4]{\frac{17}{11}}; \quad x_2 = -1 \cdot \sqrt[4]{\frac{17}{11}} = -\sqrt[4]{\frac{17}{11}}$$

$$x_3 = i \sqrt[4]{\frac{17}{11}}; \quad x_4 = -i \sqrt[4]{\frac{17}{11}}$$

3) Trinomial equations $ax^{2n} + bx^n + c = 0$

These are the types of equations: $ax^{2n} + bx^n + c = 0$, where a,b and c are real numbers (different from zero).

These equations can be solved by replacement $x^n = t \Rightarrow x^{2n} = t^2$.

Example 1. Solve the equation: $x^6 + 7x^3 - 8 = 0$

Solution:

$$\begin{aligned} x^6 + 7x^3 - 8 &= 0 \\ (x^3)^2 + 7x^3 - 8 &= 0 \quad \text{replacement: } x^3 = t \\ t^2 + 7t - 8 &= 0 \\ t_{1,2} &= \frac{-7 \pm 9}{2} \\ t_1 &= 1 \\ t_2 &= -8 \end{aligned}$$

Back in the replacement:

$$\begin{array}{ll} x^3 = 1 & \text{or} \\ x^3 - 1 = 0 & x^3 = -8 \\ (x-1)(x^2 + x + 1) = 0 & x^3 + 8 = 0 \\ x-1 = 0 \quad \vee \quad x^2 + x + 1 = 0 & x^3 + 2^3 \\ & (x+2)(x^2 - 2x + 4) = 0 \\ x_1 = 1 & x+2 = 0 \quad \vee \quad x^2 - 2x + 4 \\ & x_4 = -2 \quad x_{5,6} = \frac{2 \pm \sqrt{-12}}{2} \\ & x_{5,6} = \frac{2 \pm 2\sqrt{3}i}{2} \\ & x_{5,6} = 1 \pm \sqrt{3}i \end{array}$$

Example 2. Solve the equation: $x^8 + 17x^4 + 16 = 0$

Solution:

$$\begin{aligned} (x^4)^2 + 17x^4 + 16 &= 0 \\ x^2 + 17t + 16 &= 0 \quad \text{replacement : } x^4 = t \\ t_{1,2} &= \frac{-17 \pm 15}{2} \\ t_1 &= 16 \\ t_2 &= 1 \end{aligned}$$

$$x^4 = 16 \quad \text{or}$$

$$x^4 - 16 = 0$$

$$x^4 - 2^4 = 0$$

$$(x^2 - 2^2)(x^2 + 2^2) = 0$$

$$(x-2)(x+2)(x^2+4) = 0$$

$$x-2=0 \vee x+2=0 \vee x^2+4=0$$

$$x_1 = 2 \quad x_2 = -2 \quad x^2 = -4$$

$$x_{3,4} = \pm\sqrt{-4}$$

$$x_3 = +2i$$

$$x_4 = -2i$$

$$x^4 = 1$$

$$x^4 - 1 = 0$$

$$(x^2 - 1)(x^2 + 1) = 0$$

$$(x-1)(x+1)(x^2+1) = 0$$

$$x-1=0 \vee x+1=0 \vee x^2+1=0$$

$$x_5 = 1, \quad x_6 = -1, \quad x^2 = -1$$

$$x_{7,8} = \pm\sqrt{-1}$$

$$x_7 = +i$$

$$x_8 = -i$$

So, solutions are:

$$\{2, -2, 2i, -2i, 1, -1, +i, -i\}$$

4) Symmetrical (reciprocal) equations

$$ax^n + bx^{n-1} + cx^{n-2} + \dots + cx^2 + bx + a = 0$$

Replacement is: $x + \frac{1}{x} = t$

$$\left(x + \frac{1}{x}\right)^2 = t^2$$

$$x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = t^2$$

$$x^2 + 2 + \frac{1}{x^2} = t^2$$

$$x^2 + \frac{1}{x^2} = t^2 - 2 \rightarrow \quad \text{REMEMBER}$$

Sometimes replacement is:

$$\left(x + \frac{1}{x}\right)^3 = t^3$$

$$x^3 + 2x^2 \cdot \frac{1}{x} + 3x + \frac{1}{x^2} + \frac{1}{x^3} = t^3$$

$$x^3 + 3x + 3\frac{1}{x} + \frac{1}{x^3} = t^3$$

$$x^3 + 3\left(x + \frac{1}{x}\right) + \frac{1}{x^3} = t^3$$

$$x^3 + \frac{1}{x^3} = t^3 - 3t \rightarrow \text{REMEMBER}$$

etc. ...

Example 1. Solve the equation: $2x^4 + 3x^3 - 16x^2 + 3x + 2 = 0$

Solution:

We share the entire equation with x^2 because it is “intermediate”.

$$\frac{2x^4}{x^2} + \frac{3x^3}{x^2} - \frac{16x^2}{x^2} + \frac{3x}{x^2} + \frac{2}{x^2} = 0$$

$$2x^2 + 3x - 16 + 3 \cdot \frac{1}{x} + 2 \cdot \frac{1}{x^2} = 0 \quad \text{group members!}$$

$$2\left(x^2 + \frac{1}{x}\right) + 3\left(x + \frac{1}{x}\right) - 16 = 0 \quad \text{replacement: } x + \frac{1}{x} = t$$

$$2(t^2 - 2) + 3t - 16 = 0$$

$$2t^2 - 4 + 3t - 16 = 0$$

$$2t^2 + 3t - 20 = 0$$

$$t_{1,2} = \frac{-3 \pm \sqrt{9 + 160}}{4} = \frac{-3 \pm 13}{4}$$

$$t_1 = -4, \quad t_2 = \frac{5}{2}$$

Back in the replacement:

$$x + \frac{1}{x} = -4$$

$$x^2 + 1 = -4x$$

$$x^2 + 4x + 1 = 0$$

$$x_{1,2} = \frac{-4 \pm \sqrt{16-4}}{2}$$

$$x_1 = -2 + \sqrt{3}$$

$$x_2 = -2 - \sqrt{3}$$

$$x + \frac{1}{x} = \frac{5}{2}$$

$$2x^2 + 2 = 5x$$

$$2x^2 - 5x + 2 = 0$$

$$x_{3,4} = \frac{5 \pm \sqrt{25-16}}{4}$$

$$x_3 = 2$$

$$x_4 = \frac{1}{2}$$

i

Therefore, solutions are 2 and $\frac{1}{2}$; $-2 + \sqrt{3}$ and $-2 - \sqrt{3}$. Are they reciprocal?

It is obviously for 2 and $\frac{1}{2}$ and what is with $-2 + \sqrt{3}$ and $-2 - \sqrt{3}$?

$$\frac{-2 + \sqrt{3}}{1} = \frac{-2 + \sqrt{3}}{1} \cdot \frac{-2 - \sqrt{3}}{-2 - \sqrt{3}} = \frac{(-2)^2 - \sqrt{3}}{-2 - \sqrt{3}} = \frac{1}{-2 - \sqrt{3}}$$

Now (after rationalization), we see that it is also reciprocal.

Example 2: Solve the equation: $12x^5 + 16x^4 - 37x^3 - 37x^2 + 16x + 12 = 0$

Solution:

This is the fifth degree equation, and one solution is obviously $x = -1$. Whole equation we will share with $(x+1)$.

$$(12x^5 + 16x^4 - 37x^3 - 37x^2 + 16x + 12) : (x+1) = 12x^4 + 4x^3 - 41x^2 + 4x + 12$$

View polynomials sharing!!!

$$12x^4 + 4x^3 - 41x^2 + 4x + 12 = 0 / : x^2$$

$$12x^2 + 4x - 41 + 4 \cdot \frac{1}{x} + 12 \cdot \frac{1}{x^2} = 0$$



$$12 \left(x^2 + \frac{1}{x^2} \right) + 4 \left(x + \frac{1}{x} \right) - 41 = 0$$

$$\text{replacement : } x + \frac{1}{x} = t \Rightarrow x^2 + \frac{1}{x^2} = t^2 - 2$$

$$12(t^2 - 2) + 4t - 41 = 0$$

$$12t^2 - 24 + 4t - 41 = 0$$

$$12t^2 + 4t - 65 = 0$$

$$t_{1,2} = \frac{-4 \pm 56}{24}$$

$$t_1 = \frac{13}{6}$$

$$t_2 = -\frac{5}{2}$$

Back in the replacement:

$$x + \frac{1}{x} = \frac{13}{6}$$

and

$$x + \frac{1}{x} = -\frac{5}{2}$$

$$6x^2 - 13x + 6 = 0$$

$$2x^2 + 5x + 2 = 0$$

$$x_{1,2} = \frac{13 \pm 5}{12}$$

$$x_{3,4} = \frac{-5 \pm 3}{4}$$

$$x_1 = \frac{18}{12} = \frac{3}{2}$$

$$x_3 = -\frac{1}{2}$$

$$x_2 = \frac{8}{12} = \frac{2}{3}$$

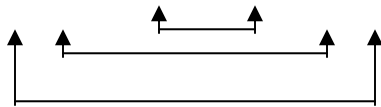
$$x_4 = -2$$

So: $\left\{ \frac{3}{2}, \frac{2}{3}, -\frac{1}{2}, -2, -1 \right\}$ are solutions!

Example 2: Solve the equation: $x^5 - 7x^4 + 16x^3 - 16x^2 + 7x - 1 = 0$

Solution:

$$x^5 - 7x^4 + 16x^3 - 16x^2 + 7x - 1 = 0$$



One solution is $x_1 = 1$. The whole equation we share with $(x-1)$

$$(x^5 - 7x^4 + 16x^3 - 16x^2 + 7x - 1) : (x-1) = x^4 - 6x^3 + 10x^2 - 6x + 1$$

$$x^4 - 6x^3 + 10x^2 - 6x + 1 = 0 \quad \text{it is symmetrical} \quad / : x^2$$

$$x^4 - 6x^3 + 10x^2 - 6x + 1 = 0$$

$$x^2 - 6x + 10 - 6 \cdot \frac{1}{x} + \frac{1}{x^2} = 0$$

$$\text{Solutions are: } x_1 = 1, x_2 = 1, x_3 = 2 + \sqrt{3}, x_4 = 2 - \sqrt{3} \quad \text{and} \quad x_5 = 1$$

