

## Irrational inequalities

We have two situations:

1)  $\sqrt{P(x)} < Q(x)$  is equivalent to:

$$P(x) > 0 \wedge Q(x) \geq 0 \wedge P(x) < Q^2(x)$$

2)  $\sqrt{P(x)} > Q(x)$  is equivalent to:

$$[P(x) \geq 0 \wedge Q(x) < 0] \vee [P(x) > Q^2(x) \wedge Q(x) \geq 0]$$

**Example 1:** Solve :  $\sqrt{x+6} < x-6$

Solution:

$$\sqrt{P(x)} < Q(x) \longrightarrow P(x) > 0 \wedge Q(x) \geq 0 \wedge P(x) < Q^2(x)$$

$$x+6 > 0 \wedge x-6 \geq 0 \wedge x+6 < (x-6)^2$$

$$x > -6 \wedge x \geq 6 \wedge x+6 < x^2 - 12x + 36$$

$$0 < x^2 - 12x + 36 - x - 6$$

$$0 < x^2 - 13x + 30$$

$$x^2 - 13x + 30 > 0$$

$$x^2 - 13x + 30 = 0$$

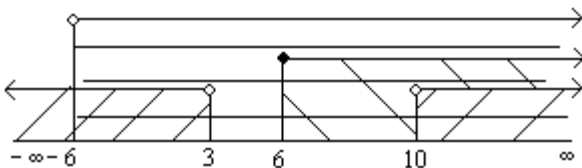
$$x_{1,2} = \frac{13 \pm \sqrt{169 - 120}}{2} = \frac{13 \pm 7}{2}$$

$$x_1 = 10$$

$$x_2 = 3$$

$$x \in (-\infty, 3) \cup (10, \infty)$$

When you solve all three inequalities, go to solution:



**Cross-section of all the three solutions is :**  $x \in (10, \infty)$

**Example 2: Solve :**  $\sqrt{x+7} > 2x-1$

$$\sqrt{P(x)} > Q(x) \longrightarrow [P(x) \geq 0 \wedge Q(x) < 0] \vee [P(x) > Q^2(x) \wedge Q(x) \geq 0]$$

$$\begin{aligned} [x+7 \geq 0 \wedge 2x-1 < 0] \\ x \geq -7 \wedge 2x < 1 \end{aligned}$$

$$\begin{aligned} x < \frac{1}{2} \\ \left[ x \geq -7 \wedge x < \frac{1}{2} \right] \end{aligned}$$

$$x \in \left[ -7, \frac{1}{2} \right)$$

$\vee$

$$[x+7 > (2x-1)^2 \wedge 2x-1 \geq 0]$$

$$x+7 > 4x^2 - 4x + 1 \quad \wedge x \geq \frac{1}{2}$$

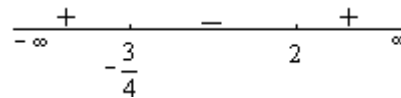
$$4x^2 - 4x + 1 - x - 7 < 0$$

$$4x^2 - 5x - 6 < 0$$

$$x_{1,2} = \frac{5 \pm 11}{8}$$

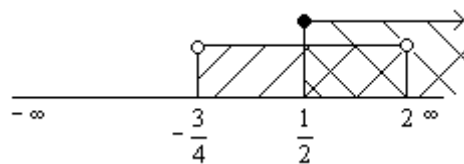
$$x_1 = 2$$

$$x_2 = -\frac{6}{8} = -\frac{3}{4}$$



$$x \in \left( -\frac{3}{4}, 2 \right)$$

$$\left[ x \in \left( -\frac{3}{4}, 2 \right) \wedge x \geq \frac{1}{2} \right]$$



$$x \in \left[ \frac{1}{2}, 2 \right)$$