

Exponential inequalities

For the exponential function is:

$$1) \quad \text{for } a > 1 \text{ is } a^{f(x)} > a^{g(x)} \Leftrightarrow f(x) > g(x)$$

$$2) \quad \text{for } 0 < a < 1 \text{ is } a^{f(x)} > a^{g(x)} \Leftrightarrow f(x) < g(x)$$

Examples:

1) Solve the inequalities:

$$a) \quad 5^{-7x+3} > 5^{-3}$$

$$b) \quad 0,35^{x-1} < 0,35^{2x+2}$$

$$c) \quad 2^{x^2-3} > 2$$

$$d) \quad 2^x < 7^x$$

Solutions:

$$a) \quad 5^{-7x+3} > 5^{-3} \rightarrow \text{as the basis is 5, the direction of inequality does not change}$$

$$-7x+3 > -3$$

$$7x > -3-3$$

$$7x > -6$$

$$x > -\frac{6}{7}$$

$$b) \quad 0,35^{x-1} < 0,35^{2x+2} \rightarrow \text{the basis is 0,3 ; } 0 < 0,3 < 1, \text{ we must face the direction of inequality}$$

$$x-1 > 2x+2$$

$$x-2x > 2+1$$

$$-x > 3$$

$$x < -3$$

$$c) 2^{x^2-3} > 2$$

$$2^{x^2-3} > 2^1$$

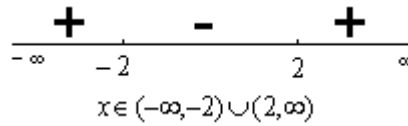
$$x^2 - 3 > 1$$

$$x^2 - 4 > 0$$

$$x_{1,2} = \frac{-0 \pm 2}{2}$$

$$x_1 = 2$$

$$x_2 = -2$$



$$d) 2^x < 7^x$$

$$\frac{2^x}{7^x} < 1$$

$$\left(\frac{2}{7}\right)^x < 1$$

$$\left(\frac{2}{7}\right)^x < \left(\frac{2}{7}\right)^0 \rightarrow \text{the basis is between 0 and 1, so, we must face the direction of inequality}$$

$$x > 0$$

2) Solve the inequalities:

$$a) 5^{2x+1} > 5^x + 4$$

$$b) 25^x < 6 \cdot 5^x - 5$$

$$c) \sqrt{9^x - 3^{x+2}} > 3^x - 9$$

Solutions:

$$a) 5^{2x+1} > 5^x + 4$$

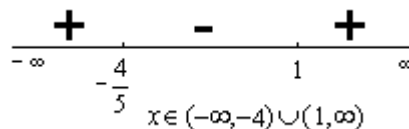
$$5^{2x} \cdot 5^1 - 5^x - 4 > 0 \rightarrow \text{replacement } 5^x = t$$

$$t^2 \cdot 5 - t - 4 > 0$$

$$t_{1,2} = \frac{1 \pm 9}{10}$$

$$t_1 = 1$$

$$t_2 = -\frac{4}{5}$$



go back in replacement $5^x = t$

$$5^x = -\frac{4}{5} \quad \text{or} \quad 5^x = 1$$

no solution $\quad x = 0 \rightarrow x \in (0, \infty)$

Now the interval $t \in (1, \infty) \longrightarrow$ transformed into $x \in (0, \infty)$

$x \in (0, \infty)$ is solution!

b) $25^x < 6 \cdot 5^x - 5$

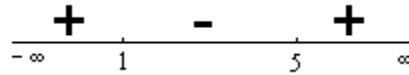
$5^{2x} - 6 \cdot 5^x + 5 < 0 \rightarrow$ replacement $5^x = t$

$t^2 - 6t + 5 < 0$

$t_{1,2} = \frac{6 \pm 4}{2}$

$t_1 = 5$

$t_2 = 1$



$t \in (1, 5)$, go back in replacement $5^x = t$

$5^x = 1 \quad \text{or} \quad 5^x = 5$

$x = 0 \quad \text{or} \quad x = 1$

$x \in (0, 1)$ is the final solution.

c) $\sqrt{9^x - 3^x \cdot 3^2} > 3^x - 9$

$\sqrt{3^{2x} - 3^x \cdot 9} > 3^x - 9 \rightarrow$ replacement $3^x = t$

$\sqrt{t^2 - 9t} > t - 9$

$[t^2 - 9t \geq 0 \wedge t - 9 < 0]$

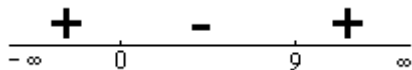
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$[t^2 - 9t \geq (t - 9)^2 \wedge t - 9 \geq 0]$

$t_{1,2} = \frac{9 \pm 9}{2}$

$t_1 = 0 \quad t < 9$

$t_2 = 9$



$t \in (-\infty, 0] \cup [9, \infty)$

$t \in (-\infty, 0]$

this interval "does not work" because it is $3^x = t$

$t^2 - 9t > t^2 - 18t + 81$

$-9t + 18t > 81$

$9t > 81$

$t > 9$

So: $t > 9$

$3^x > 9$

$3^x > 3^2$

$x > 2$

the final solution

