Systems of linear equations

System of two linear equations with two unknown x and y:

$$a_1x+b_1y=c_1$$
$$a_2x+b_2y=c_2$$

This is the so-called "simple" system to which we can always get with equivalent transformations (described in the equations)

Here are $a_1, a_2, b_1, b_2, c_1, c_2$ the real numbers (and sometimes parameters).

Solution is couple (x_0, y_0):

$$a_1x_0+b_1y_0=c_1$$
$$a_2x_0+b_2y_0=c_2$$

Systems can be solved using several methods: replacement, contrary coefficients, Gauss, with the determinants, matrix, etc.

For us is most important to correctly solve the task (the problem) and we will try to learn you that.

Mention only, that the system can have: a unique solution, infinite much solutions (neutral system) or that there is no solution (impossible system).

Example 1: Solve the system:

$$2x+3y=7$$
$$3x-6y=7$$

$$2x+3y=7 \quad \text{/*2}$$
$$3x-6y=7$$

Make the same numbers and opposite sign(in fron of x or y), and then these two equations gather . Therefore, we will multiply the first equation by 2

$$+\begin{cases}4x+6y=14\\3x-6y=7\end{cases}$$

When you find one solution, go to one of the equation (any) to find another solution.

$$7x=21$$
$$x=\frac{21}{7}$$
$$x=3$$

$$2x+3y=7$$

$$2x+y=7$$
$$2\cdot 3+3y=7$$
$$6+3y=7$$
$$3y=7-6$$
$$3y=1$$
$$y=\frac{1}{3}$$

Here is a unique solution: $(x,y)=\left(3,\frac{1}{3}\right)$

Example 2: Solve the system : $$\begin{array}{r} 5x + y = -1 \\ -10x - 2y = 2 \end{array}$$

$$\begin{array}{l} 5x + y = -1 /(-2) \\ -10x - 2y = 2 \end{array}$$ Multiply the first equation with (-2)

$$\begin{array}{r} +10x + 2y = -2 \\ -10x - 2y = 2 \\ \hline 0 = 0 \end{array}$$

This tells us that the system has a lot of Infinite solutions. To describe that "solutions", from one of the equation express x (or y), of course, what is easier.

$$\begin{array}{l} 5x + y = -1 \\ y = -1 - 5x \end{array}$$

Now, the solutions are: $(x, y) = (x, -1 - 5x)$ $\quad x \in R$

Example 3: Solve the system : $$\begin{array}{r} 2x + 3y = 4 \\ -2x - 3y = 5 \end{array}$$

$$\begin{array}{r} 2x + 3y = 4 \\ -2x - 3y = 5 \\ \hline 0 = 9 \end{array}$$ Gather them immediately

In this situation we say that system is impossible, and no solutions.

Example 4: Solve the system : $$\begin{array}{l} \frac{5x-1}{6} + \frac{3y-1}{10} = 3 \\ \frac{11-x}{6} + \frac{11+y}{4} = 3 \end{array}$$

$$\frac{5x-1}{6}+\frac{3y-1}{10}=3..../\cdot 30$$
$$\frac{11-x}{6}+\frac{11+y}{4}=3..../\cdot 12$$

$$5(5x-1)+3(3y-1)=90$$
$$2(11-x)+3(11+y)=36$$

$$25x-5+9y-3=90$$
$$22-2x+33+3y=36$$

$$25x+9y=90+5+3$$
$$-2x+3y=-19..../\cdot(-3)$$

$$\left.\begin{aligned}25x+9y&=98\\6x-9y&=57\end{aligned}\right\}+$$

$$31x=155$$
$$x=5$$

Let's go back now in one of the equation for a simple system.

$$-2x+3y=-19$$
$$-2\cdot 5+3y=-19$$
$$3y=-19+10$$
$$3y=-9$$
$$y=-3$$

$(x,y)=(5,-3)$ is solution

Example 5: Solve the system : $\frac{14}{x}+\frac{24}{y}=10$

$\frac{7}{x}-\frac{18}{y}=-5$ We observe that unknown are in divisor.

In such situation it is best to take replacement:

$\frac{1}{x}=a$ and $\frac{1}{y}=b$

$$14\cdot\frac{1}{x}+24\cdot\frac{1}{y}=10$$
$$7\cdot\frac{1}{x}-18\cdot\frac{1}{y}=-5$$

$\longrightarrow$

$$14a+24b=10$$
$$7a-18b=-5$$

$$14a + 24b = 10$$
$$7a - 18b = -5 / (-2)$$

$$\left.\begin{aligned}14a + 24b &= 10\\ -14a + 36b &= 10\end{aligned}\right\}+$$

$$60b = 20$$
$$b = \frac{20}{60}$$
$$b = \frac{1}{3}$$

$$7a - 18b = -5$$
$$7a - 18 \cdot \frac{1}{3} = -5$$
$$7a - 6 = -5$$
$$7a = -5 + 6$$
$$7a = 1$$
$$a = \frac{1}{7}$$

Back in replacement to find x and y.

$$\frac{1}{x} = a, \qquad \frac{1}{y} = b$$
$$\frac{1}{x} = \frac{1}{7}, \qquad \frac{1}{y} = \frac{1}{3}$$
$$x = 7 \qquad y = 3$$
$$\longrightarrow (x, y) = (7,3)$$

Example 6: Solve the system : $\begin{aligned}ax - 9y &= 14a\\ 2ax + 3y &= 7a\end{aligned}$

Here we note that there is parameter **a.** Careful!

$$ax - 9y = 14a$$
$$2ax + 3y = 7a$$

$$ax - 9y = 14a$$
$$2ax + 3y = 7a \ldots / \cdot 3$$

$$\left.\begin{aligned}ax - 9y &= 14a\\ +6ax + 9y &= 21a\end{aligned}\right\}+$$

$$7ax = 35a$$
$$x = \frac{35a}{7a} \qquad a \neq 0 \text{ ,the condition}$$
$$x = 5a$$

$ax - 9y = 14a$
$5a - 9y = 14a$
$-9y = 14a - 5a$
$-9y = 9a$
$y = -a$

Solutions are $(x, y) = (5a, -a)$ $\quad a \neq 0$

What happens if $a = 0$?

Change value $a = 0$ to the initial system:

$0 \cdot x - 9y = 0$ Here you can see that $y = 0$ and x can be any number.
$0 \cdot x + 3y = 0$

$y = 0 \qquad x \in R$

System three equations with three unknown

7) Solve the system:

$$x + 2y - 5z = 6$$
$$-2x + y + 2z = 5$$
$$-3x + 3y - 4z = 8$$

$$\begin{aligned} x + 2y - 5z &= 6 \\ -2x + y + 2z &= 5 \\ -3x + 3y - 4z &= 8 \end{aligned}$$

The easiest is to get the I and II, I and III equation and free of one unknown.
So, we have 2 equations with 2 unknowns.

$$\begin{aligned} x + 2y - 5z &= 6 \,/\cdot 2 \\ -2x + y + 2z &= 5 \end{aligned}$$

$$\left.\begin{aligned} 2x + 4y - 10z &= 12 \\ -2x + y + 2z &= 5 \end{aligned}\right\} +$$

$$5y - 8z = 17$$

$$\begin{aligned} x + 2y - 5z &= 6 \,/\cdot 3 \\ -3x + 3y - 4z &= 8 \end{aligned}$$

$$\left.\begin{aligned} 3x + 6y - 15z &= 18 \\ -3x + 3y - 4z &= 8 \end{aligned}\right\} +$$

$$9y - 19z = 26$$

Now take these two equations and find unknown y and z.

$$5y-8z=17\ /\cdot 9$$
$$9y-19z=26/\cdot(-5)$$

$$\left.\begin{array}{l}45y-72z=153\\ -45y+95z=-130\end{array}\right\}+$$

$$23z=23$$
$$z=1$$

$$5y-8z=17$$
$$5y-8=17$$
$$5y=25$$
$$y=5$$

When you find 2 unknown,go back in one of the first three equations, (any)

$$x+2y-5z=6$$
$$x+2\cdot 5-5\cdot 1=6$$
$$x+10-5=6$$
$$x=6-10+5$$
$$x=1$$

$$(x,y,z)=(1,5,1)$$

8) Solve the system:

$$2x-3y+z=-9$$
$$5x+y-2z=12$$
$$x-2y-3z=1$$

Now is easier to get rid of the unknown z.

$$2x-3y+z=-9/\cdot 2 \qquad 2x-3y+z=-9/\cdot 3$$
$$5x+y-2z=12 \qquad x-2y-3z=1$$

$$4a-6y+2z=-18 \qquad 6x-9y+3z=-27$$
$$5x+y-2z=12 \qquad x-2y-3z=1$$

$$9x-5y=-6 \qquad 7x-11y=-26$$

Take these two equations and find x and y.

$$9x-5y=-6\,/\cdot(-7)$$
$$7x-11y=-26/\cdot 9$$

$$-63x+35y=42$$
$$63x-99y=-234$$

$$-64y=-192$$
$$y=3$$

$$x-2y-3z=1$$
$$1-2\cdot 3-3z=1$$
$$1-6-3z=1$$
$$-3z=1+5$$
$$-3z=6$$
$$z=-2$$

$$(x,y,z)=(1,3,-2)$$