

## LINEAR EQUATIONS IN ONE VARIABLE

A common form of linear equation in one variable is :

$$a \cdot x = b$$

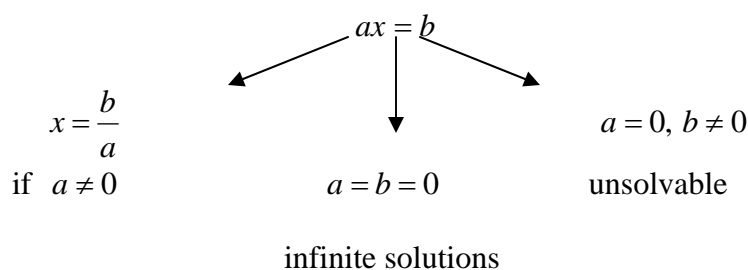
a and b are real numbers.

A solution  $x_0$  is every real number for which is :

$$ax_0 = b$$

If we have:  $A \cdot B = 0$  or  $A \cdot B \cdot C = 0$  then:

$$\begin{aligned} A \cdot B = 0 &\Leftrightarrow A = 0 \vee B = 0 \\ A \cdot B \cdot C = 0 &\Leftrightarrow A = 0 \vee B = 0 \vee C = 0 \end{aligned}$$



Technique is to add, subtract, multiplu or divide both sides of the equation by the same number in order to isolate the variable on one side of the equation!

### EXAMPLES

1) Solve the equation:

$$9 - 2x = 5x + 2$$

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$$-2x - 5x = +2 - 9$$

$$-7x = -7$$

$$x = \frac{-7}{-7}$$

$$x = 1$$

2) Solve the equation:

$$3(2 - 3x) + 4(6x - 11) = 10 - x$$

$$6 - 9x + 24x - 44 = 10 - x$$

$$-9x + 24x + x = 10 - 6 + 44$$

$$16x = 48$$

$$x = \frac{48}{16}$$

$$x = 3$$

3) Solve the equation :  $\frac{y-5}{7} + 2 = \frac{2y-3}{2} - \frac{6y+5}{14}$

$$\frac{y-5}{7} + 2 = \frac{2y-3}{2} - \frac{6y+5}{14} \cdot 14$$

$$2(y-5) + 28 = 7(2y-3) - 1(6y+5)$$

$$2y - 10 + 28 = 14y - 21 - 6y - 5$$

$$2y - 14y + 6y = -21 - 5 + 10 - 28$$

$$-6y = -44$$

$$y = \frac{-44}{-6}$$

$$y = +\frac{22}{3}$$

4) Solve the equation :  $(x+3)^2 - (x-4)^2 = 2x - 13$

$$(x+3)^2 - (x-4)^2 = 2x - 13$$

$$(x^2 + 6x - 9) - (x^2 - 8x + 16) = 2x - 13$$

$$x^2 + 6x + 9 - x^2 + 8x - 16 = 2x - 13$$

$$6x + 8x - 2x = -13 - 9 + 16$$

$$12x = -6$$

$$x = \frac{-6}{12}$$

$$x = -\frac{1}{2}$$

5) Solve the equation :  $\frac{2}{x-2} = \frac{1}{x+3}$

pay attention: avalanche:  $x-2 \neq 0 \Rightarrow x \neq 2$  and  $x+3 \neq 0 \Rightarrow x \neq -3$

$$\frac{2}{x-2} = \frac{1}{x+3}$$

$$2(x+3) = 1 \cdot (x-2)$$

$$2x + 6 = x - 2$$

$$2x - x = -2 - 6$$

$$x = -8$$

6) Solve the equation :  $\frac{x+5}{3x-6} = \frac{1}{2} + \frac{2x-3}{2x-4}$

Avalanche:  $3x-6 \neq 0$  and  $2x-4 \neq 0$   $x \neq 2$

$$\frac{x+5}{3x-6} = \frac{1}{2} + \frac{2x-3}{2x-4}$$

$$\frac{x+5}{3(x-2)} = \frac{1}{2} + \frac{2x-3}{2(x-2)} \quad / \cdot 6(x-2)$$

$$2(x+5) = 3(x-2) + 3(2x-3)$$

$$2x+10 = 3x-6+6x-9$$

$$2x-3x-6x = -6-9-10$$

$$-7x = -25$$

$$x = \frac{25}{7}$$

7) Solve the equation :  $\frac{2x-1}{2x+1} + \frac{8}{4x^2-1} = \frac{2x+1}{2x-1}$

$$\frac{2x-1}{2x+1} + \frac{8}{4x^2-1} = \frac{2x+1}{2x-1}$$

$$\frac{2x-1}{2x+1} + \frac{8}{(2x-1)(2x+1)} = \frac{2x+1}{2x-1} \quad / \cdot (2x-1)(2x+1)$$

$$(2x-1)^2 + 8 = (2x+1)^2$$

$$4x^2 - 4x + 1 + 8 = 4x^2 + 4x + 1$$

$$4x^2 - 4x - 4x^2 - 4x = 1 - 1 - 8$$

$$-8x = -8$$

$$x = 1$$

$$\begin{array}{ll} 2x+1 \neq 0 & 2x-1 \neq 0 \\ 2x \neq -1 & 2x \neq 1 \\ x \neq -\frac{1}{2} & x \neq \frac{1}{2} \end{array}$$

8) Solve the equation :  $|5x-1| + x = 2$

$$|\otimes| = \begin{cases} \otimes, \otimes \geq 0 \\ -\otimes, \otimes < 0 \end{cases} \quad |5x-1| = \begin{cases} 5x-1, \text{if } 5x-1 \geq 0 \\ -(5x-1), \text{if } 5x-1 < 0 \end{cases} = \begin{cases} 5x-1, & x \geq \frac{1}{5} \\ -(5x-1), & x < \frac{1}{5} \end{cases}$$

We have two equations:

$$\underline{\text{if } x \geq \frac{1}{5}}$$

$$5x-1+x=2$$

$$6x=2+1$$

$$6x=3$$

$$x = \frac{3}{6}$$

$$x = \frac{1}{2}$$

$$\underline{\text{if } x < \frac{1}{5}}$$

$$-(5x-1)+x=2$$

$$-5x+1+x=2$$

$$-4x=2-1$$

$$-4x=1$$

$$x = -\frac{1}{4}$$

9) Solve the equation :  $|x-4| - |2x+3| = 2$

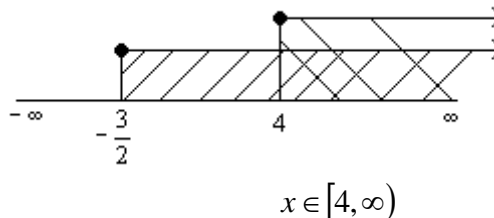
$$|x-4| = \begin{cases} x-4, & x-4 \geq 0 \\ -(x-4), & x-4 < 0 \end{cases} = \begin{cases} x-4, & x \geq 4 \text{ I avalanche} \\ -(x-4), & x < 4 \text{ II} \end{cases}$$

$$|2x+3| = \begin{cases} 2x+3, & 2x+3 \geq 0 \\ -(2x+3), & 2x+3 < 0 \end{cases} = \begin{cases} 2x+3, & x \geq -\frac{3}{2} \text{ III} \\ -(2x+3), & x < -\frac{3}{2} \text{ IV} \end{cases}$$

We will divide task in 4 parts:

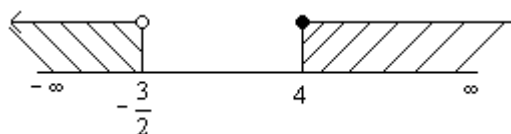
I and III

$$\begin{aligned} x \geq 4 \text{ i } x \geq -\frac{3}{2} \\ (x-4) - (2x+3) &= 2 \\ x-4-2x+3 &= 2 \\ -x &= 2+4+3 \\ -x &= 9 \\ x &= -9 \text{ is not a solution because } x \in [4, \infty) \end{aligned}$$



I and IV

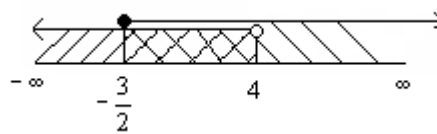
$$x \geq 4, x < -\frac{3}{2}$$



unsolvable

II and III

$$\begin{aligned} x < 4 \text{ i } x \geq -\frac{3}{2} \\ -(x-4) - (2x+3) &= 2 \\ -x+4-2x-3 &= 2 \\ -3x &= 2+3-4 \\ -3x &= 1 \\ x &= -\frac{1}{3} \end{aligned}$$



$$-\frac{1}{3} \in \left[-\frac{3}{2}, 4\right)$$

II and IV

$$x < 4, \text{ i } x < -\frac{3}{2}$$

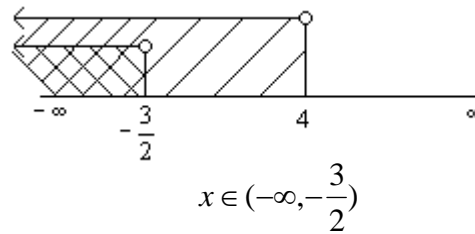
$$-(x-4) + (2x+3) = 2$$

$$-x + 4 + 2x + 3 = 2$$

$$x = 2 - 4 - 3$$

$$x = -5$$

$$-5 \in \left(-\infty, -\frac{3}{2}\right)$$



9) Solve the equation :  $mx - 3m = 1 + 5x$   $m \in R$

$$mx - 3m = 1 + 5x$$

$$mx - 5x = 1 + 3m$$

$$x(m - 5) = 1 + 3m$$

$$x = \frac{1 + 3m}{m - 5}$$

If  $m = 5 \Rightarrow x = \frac{1 + 3 \cdot 5}{0} \Rightarrow$  unsolvable

If  $m \neq 5 \Rightarrow x = \frac{1 + 3m}{m - 5} \Rightarrow$  undetermined, because  $m \in R$ ,  $\infty$  solutions

10) Solve the equation :  $2ax - a + 4 = 8a + 7 - 5x$

$$2ax - a + 4 = 8a + 7 - 5x$$

$$2ax + 5x = 8a + 7 + a - 4$$

$$x(2a + 5) = 9a + 3$$

$$x = \frac{9a + 3}{2a + 5}$$

If  $2a + 5 = 0 \Rightarrow a = -\frac{5}{2}$  unsolvable

If  $2a + 5 \neq 0 \Rightarrow$  undetermined,  $\infty$  solutions

Equations have big application in “ problem” tasks. It is important to study the text, if there is a need , make a sketch of problem and find a connection between data.

11) Father has 43 year and son 18. For how many years will father be two times older than son?

Mark with x number of years that have to pass.

Father – 43 years

Son- 18 years

How years are passing for father and for son...we have:

Father –  $43+x$  years

Son -  $18+x$  years

$$2 \cdot (18 + x) = 43 + x$$

$$36 + 2x = 43 + x$$

$$2x - x = 43 - 36$$

$$x = 7$$

12) Student read one day  $\frac{1}{4}$  of the book, second day  $\frac{2}{3}$  from the rest of the book, third day last 40 pages.

How many pages has that book?

Mark with x number of pages.

$$\frac{1}{4}x + \frac{2}{3} \cdot \frac{3}{4}x + 40 = x$$

$$\frac{1}{4}x + \frac{2}{4}x + 40 = x$$

$$\frac{3}{4}x + 40 = x$$

$$x - \frac{3}{4}x = 40$$

$$\frac{1}{4}x = 40$$

$$x = 160$$

13) One worker can finish a job in 9, and second in 12 days. If there is a third worker , they will finish the job in 4 days. For how long will the third worker finish the job alone?

x- is time in wich third worker finish a job.

How do we think?

If the first finish a job in 9 days, then for 1 day he will do  $\frac{1}{9}$  of job.

Similar , second worker will do  $\frac{1}{12}$  of job and third  $\frac{1}{x}$  of job.

It means that they together for one day do  $\frac{1}{9} + \frac{1}{12} + \frac{1}{x}$  part of job. They are working 4 days, so:

$$\left(\frac{1}{9} + \frac{1}{12} + \frac{1}{x}\right) \cdot 4 = 1$$

$$\frac{1}{9} + \frac{1}{12} + \frac{1}{x} = 1/4 \cdot 36x$$

$$16x + 12x + 144 = 36x$$

$$28x - 36x = -144$$

$$-12x = -144$$

$$x = 12$$

The third worker will finish the job for 12 days!