Euler replacaments for integrals

We use this replacements on type of integrals: $\int R[x,\sqrt{ax^2+bx+c}]dx$

1) If a>0 then replacement is: $\sqrt{ax^2+bx+c} = \pm\sqrt{a}$ x+t

2) If c>0 then replacement is: $\sqrt{ax^2+bx+c}$ = xt + $\sqrt{c}$

3) If x_1 and x_2 are solutions of square equations, replacement is: $\sqrt{ax^2+bx+c}$ =a(x-x_1)t or

$\sqrt{ax^2+bx+c}$ =a(x- x_2)t

Examples:

1. $\int \frac{dx}{x+\sqrt{x^2+x+1}}$, here we use 1. replacement $\sqrt{ax^2+bx+c} = \pm\sqrt{a}$ x+t or

a = 1, b =1, c=1 $\sqrt{x^2+x+1} = \pm$ x + t . Is it smarter to take the + or - ?

Let expression : $x+\sqrt{x^2+x+1}$, it is smarter to take – x + t, because in this case we have: x – x + t = t…..

2. $\int \frac{dx}{x+\sqrt{x^2-5x+6}}$, think what is the best replacement…

3. $\int \frac{(1-\sqrt{x^2+x+1})^2}{x^2\sqrt{x^2+x+1}}dx$, think what is the best replacement…

A little advice:

Euler's integral similar, where we have * instead of + or -, for example $\int \frac{dx}{x\sqrt{x^2+x+1}}$, is better to do with replacement : x =$\frac{1}{t}$.