DOUBLE INTEGRALS

1. MAKING DIRECTLY

If the area is determined with inequality D:

$$\begin{cases} a \le x \le b \\ y_1(x) \le y \le y_2(x) \end{cases} \quad \text{then:} \quad \iint_D z(x,y)dxdy = \int_a^b dx \int_{y_1(x)}^{y_2(x)} z(x,y)dy$$

If the area is determined with inequality D:

$$\begin{cases} x_1(y) \le x \le x_2(y) \\ c \le y \le d \end{cases} \quad \text{then :} \quad \iint_D z(x,y)dxdy = \int_c^d dy \int_{x_1(y)}^{x_2(y)} z(x,y)dx$$

2. GENERAL COORDINATES

If the x = x (u, v) and y = y (u, v),then

$$J=\frac{D(x,y)}{D(u,v)} \ne 0 \qquad \text{Then the formula is: } \iint_D z(x,y)dxdy = \iint_{D`} z[x(u,v),y(u,v)]\left|J\right|dudv$$

3 POLAR COORDINATES

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \\ \left|J\right| = r \end{cases} \quad \text{then:} \quad \iint_D z(x,y)dxdy = \iint_{D`} z(r\cos\varphi, r\sin\varphi)\left|J\right|drd\varphi = \int_{\varphi_1}^{\varphi_2} d\varphi \int_0^r z(r\cos\varphi, r\sin\varphi)rdr$$

4.LINEAR COORDINATES

$$\begin{cases} x = ar\cos\varphi \\ y = br\sin\varphi \\ \left|J\right| = abr \end{cases} \quad \text{then:} \quad \iint_D z(x,y)dxdy = \int_{\varphi_1}^{\varphi_2} d\varphi \int_0^r z(ar\cos\varphi, br\sin\varphi)abrdr$$

APPLICATION of DOUBLE INTEGRALS

i) Area

If z = z (x, y) and $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$ then: $A=\iint_D \sqrt{1+p^2+q^2}\,dxdy$

If x = x (u, v) and y = y (u,v) it is:

$$A=\iint_D \sqrt{EG-F^2}\,dudv$$ where are:

$$E = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2$$

$$G = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2$$

$$F = \frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial y}{\partial u}\frac{\partial y}{\partial v} + \frac{\partial z}{\partial u}\frac{\partial z}{\partial v}$$

ii) Volume

If we have z = z (x, y),

$$V = \iint_D z(x,y)\,dxdy$$